Robot Motion Planning for Multiple Moving Objects Interception with constant acceleration

J. A. Flores Campos, E. C. Dean León, C. Palacios Montufar

Abstract: This paper presents a novel approach to on-line, robot-motion planning for multiple moving-objects interception with constant acceleration. The position, velocity and acceleration of the free particles are used as kinematics information. This information is used to obtain the multiple interception trajectory. The polynomial interpolation technique is implemented in a 2D robot with to follow the follower-trajectory generated. The implementation of the proposed technique is ilustrated via a simulation example. It consists of the 2D multiple interception of two objects moving along a well-known parabolic trajectory via a robot interceptor.

Keywords: Robot planning, multiple interception, trajectory.

Introduction

The problem addressed in this paper is the multiple interception of two or more free particles moving with constant acceleration. A slow-maneuvering target moves on a continuous path with a relatively constant acceleration. The state (position and velocity) of the objects as a function of time is obtained through a vision system. The moving objects are assumed to stay within the robot´s workspace for a limited time.

The multiple interception task is defined as "approaching to multiple moving objects while matching their locations and velocities in the shortest possible time."[1]. Prediction, planning, and execution (PPE) methods are well suited for intercepting objects traveling along predictable trajectories, when using PPE technique, the robot is directly sent to an anticipated rendezvous point on the target´s predicted trajectory [4-7].

Juan Alejandro Flores Campos.
Emmanuel Carlos Dean León.
Academia de Mecatrónica. UPIITA–IPN. Av. IPN No. 2580. Col. Barrio la Laguna Ticomán, Delg. Gustavo A. Madero, C.P. 07340. México, D.F. jaflores@ipn.mx.

Cándido Palacios Montufar, SEPI-ESIME-ZACATENCO-IPN, Av. Instituto Politécnico Nacional, México, D.F.

Problem definition: The n particles: TFL_1 (Trajectory of the Free Particle 1), TFL_2 (Trajectory of the Free Particle 2), TFL_n (Trajectory of the Free Particle ***n***) and *TFP* (Trajectory of the Following Particle). TFL_i, i=1,..,n describe well-known trajectories. In time $t=t_0=0$ s, begin their motion from the initial position (x_1, y_1, z_1), (x_2, y_2, z_2),…, (x_n, y_n, z_n), respectively. *TFP* is a trajectory to be calculated mathematically. *TFP* begins its motion at the initial position (ax,ay,az) in the delayed time $t=t_1$ where $t_0<t_1\leq T^*$. T^* is a bounded time obtained from the limit job space of the robot [2-3].

The function *TFP* have to intercept TFL_1 in the instant of time t_2 and to equal the direction of its trajectory in the interval of time $\Delta t_2=t_3-t_2$ then, have to intercept TFL_2 in the instant of time t_4 and to equal the direction of its trajectory in the interval of time $\Delta t_4=t_5-t_4$, thus, successively until intercepting to TFL_n equal the direction of its trajectory in the interval of time $\Delta t_n=t_{n+1}-t_n$. see fig (1).

Methodology of multiple 3D interception of well-known trajectories:

A methodology to solve the problem to calculate mathematically the following-trajectory *TFP* that intercepts the trajectories described by *TFLi*, i=1,…,n in the defined times t_2, t_4,…, t_n, is presented.

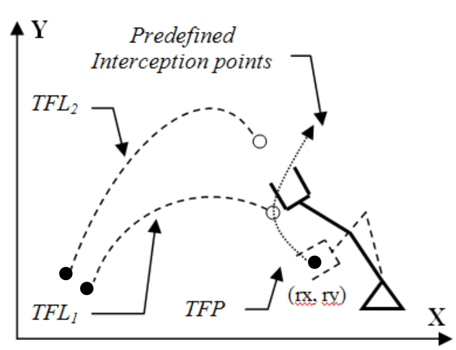

Fig. 1. Trajectories TFL_1, TFL_2 and *TFP*.

The function *TFLi* (Trajectory of the Free Particle *i*): The ***n*** trajectories generated by the particles *TFLi*, *i*=1,…,*n* can be represented in the parametric form, being ***t*** the parameter [2-3], see (1).

$$TFL_i = (X_i(t), Y_i(t), Z_i(t))^T, \qquad i = 1, \ldots, n \qquad (1)$$

The components of eq.(1): $X_1(t)$, $Y_1(t)$, $Z_1(t)$,…, $X_n(t)$, $Y_n(t)$, $Z_n(t)$ are functions formed with the next form with constant acceleration. See (2).

$$TFL_n \begin{cases} X_n(t) = x_{0n} + V_{x0n} t + \frac{1}{2} a_{xn} t \\ Y_n(t) = y_{0n} + V_{y0n} t + \frac{1}{2} a_{yn} t \\ Z_n(t) = z_{0n} + V_{z0n} t + \frac{1}{2} a_{zn} t \end{cases} \qquad (2)$$

Where x_{0n}, y_{0n} *and* z_{0n} are the inicial position of the n^{th} particle. In the same way, Vx_{0n}, Vy_{0n} *and* Vz_{0n} are the inicial velocity. The constant acceleration components are: $a_{xn}, a_{yn}, a_{zn},$ The acceleration goverments the trayectory topology of the particle. This information can be obtained using a vision system.

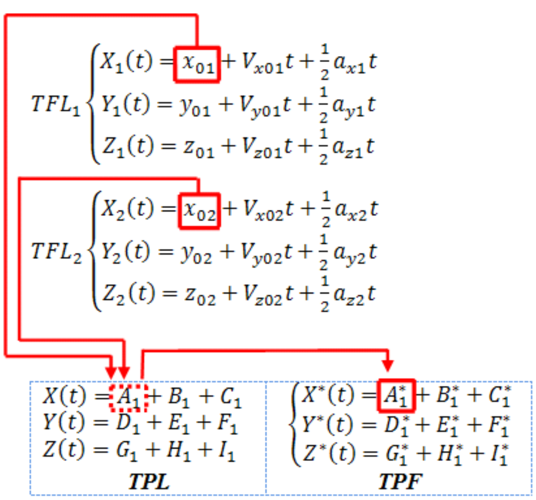

Fig. 2. Grouping the terms of the function TFL.

The fig. 2, shows the way how can represent the components in the correspondent term in the function TFL. In the same way can be obtained for the velocity and acceleration. In the compact form: see (3).

$$TFL_i \begin{cases} X_i(t) = \sum_{j=1}^{k} Tx_{ij}(t) \\ Y_i(t) = \sum_{j=1}^{l} Ty_{ij}(t) \\ Z_i(t) = \sum_{j=1}^{m} Tz_{ij}(t) \end{cases} \qquad (3)$$

Where: *k*, *l* and *m* are the total number of terms of the functions $X_i(t)$, $Y_i(t)$ *and* $Z_i(t)$, respectively.

The function *TFP* (Trajectory of the Following-Particle): To generate the following-trajectory *TFP* the following functions are proposed, position, velocity and acceleration, respectively, see (4):

$$\begin{aligned} TFP &= (X^*(t), Y^*(t), Z^*(t)) \\ T\dot{F}P &= (\dot{X}^*(t), \dot{Y}^*(t), \dot{Z}^*(t)) \\ T\ddot{F}P &= (\ddot{X}^*(t), \ddot{Y}^*(t), \ddot{Z}^*(t)) \end{aligned} \qquad (4)$$

The components of the following-function *TFP* can be expressed, as (5), (6) and (7): Could be represented as follows:

$$TFP = \begin{cases} X^*(t) = A_1 + B_1 + C_1 \\ Y^*(t) = D_1 + E_1 + F_1 \\ Z^*(t) = G_1 + H_1 + I_1 \end{cases} \qquad (5)$$

$$T\dot{F}P = \begin{cases} \dot{X}^*(t) = \dot{A}_1 + \dot{B}_1 + \dot{C}_1 \\ \dot{Y}^*(t) = \dot{D}_1 + \dot{E}_1 + \dot{F}_1 \\ \dot{Z}^*(t) = \dot{G}_1 + \dot{H}_1 + \dot{I}_1 \end{cases} \qquad (6)$$

$$T\ddot{F}P = \begin{cases} \ddot{X}^*(t) = \ddot{A}_1 + \ddot{B}_1 + \ddot{C}_1 \\ \ddot{Y}^*(t) = \ddot{D}_1 + \ddot{E}_1 + \ddot{F}_1 \\ \ddot{Z}^*(t) = \ddot{G}_1 + \ddot{H}_1 + \ddot{I}_1 \end{cases} \qquad (7)$$

Representating in the compact form.

$$\begin{aligned} X^*(t) &= \sum_{j=1}^{k} Tx_j^*(t), \\ Y^*(t) &= \sum_{j=1}^{l} Ty_j^*(t), \\ Z^*(t) &= \sum_{j=1}^{m} Tz_j^*(t) \end{aligned} \qquad (8)$$

Estimating the terms $Tx^*_j(t), Ty^*_j(t), Tz^*_j(t)$: For each term $Tx_{ij}(t), Ty_{ij}(t)$ and $Tz_{ij}(t)$ of the functions $X_i(t)$, $Y_i(t)$ and $Z_i(t)$ shown in (2), the functions $Tx^*_j(t)$, $Ty^*_j(t)$ and $Tz^*_j(t)$ are proposed, respectively, that intercepts each one of the terms.

In other words, it is necessary to know the kinematics information of each term $Tx_{ij}(t), Ty_{ij}(t)$ and $Tz_{ij}(t)$. With this information we generate the functions: $X_i(t), Y_i(t)$ and $Z_i(t)$ of the trajectories described by free particles *TFLi*, $i=1,\ldots,n$ in order to calculate the terms $Tx^*_j(t)$, $Ty^*_j(t)$ and $Tz^*_j(t)$ they have the form of the following-trajectory *TFP*. Without loss of generality, as example to illustrate the methodology we only develop $Tx^*_j(t)$ with j=0, that is; $Tx^*_0(t)$, the information of $Tx_{ij}(t)$, $i=0,\ldots,n$ is used, that is, the terms: $Tx_{00}(t)$, $Tx_{10}(t)$, … , $Tx_{n0}(t)$, are expressed in (2) and illustrated in fig. 3.

We take the first term $Tx_{i0}(t)$, $i=0,\ldots,n$ of each function $X_i(t)$, $i=0,\ldots,n$, the terms are shown in fig. 3.

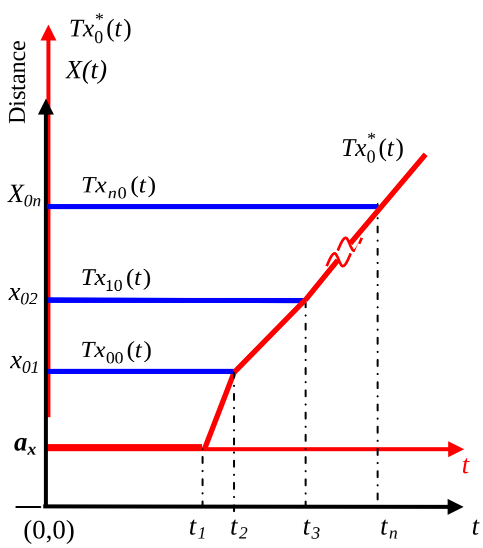

Fig.3 Estimation of the function $Tx^*_0(t)$ and $\Delta t_2=t_3-t_2$ and $\Delta t_4=t_5-t_4$ intervals of time.

We can observe that $Tx^*_0(t)$ intercepts $Tx_{00}(t)$ in the time t_2, and to equal the direction of its trajectory in the interval of time $\Delta t_2=t_3-t_2$. Then, it intercepts $Tx_{10}(t)$ in the time t_4, and to equal the direction of its trajectory in the interval of time $\Delta t_4=t_5-t_4$.

Finally, it intercepts to $Tx_{n0}(t)$ in the time t_n and to equal the direction of its trajectory in the interval of time $\Delta t_n=t_{n+1}-t_n$. To calculate $Ty^*_0(t)$ is also required to know the functions $Ty_{00}(t)$, $Ty_{10}(t)$, … , $Ty_{n0}(t)$, which are the terms of $Y_i(t)$. $Tz^*_0(t)$ can be obtained in the same way. In the table 1, the information of the terms: $Tx_{ij}(t)$, $Ty_{ij}(t)$ and $Tz_{ij}(t)$ necessary to calculate $Tx^*_j(t)$, $Ty^*_j(t)$ and $Tz^*_j(t)$ respectively, are presented.

Table 1. Information to estimate $Tx^*_j(t)$

j	$Tx^*_j(t)$	Terms: $Tx_{ij}(t)$, $i=0,\ldots,n$
0	$Tx^*_0(t)$	$Tx_{00}(t), Tx_{10}(t), Tx_{20}(t), \ldots, Tx_{n0}(t)$
1	$Tx^*_1(t)$	$Tx_{01}(t), Tx_{11}(t), Tx_{21}(t), \ldots, Tx_{n1}(t)$
2	$Tx^*_2(t)$	$Tx_{02}(t), Tx_{12}(t), Tx_{22}(t), \ldots, Tx_{n2}(t)$
..	…….	
k	$Tx^*_k(t)$	$Tx_{0k}(t), Tx_{1k}(t), Tx_{2k}(t), \ldots, Tx_{nk}(t)$

Form of the functions $Tx^*_j(t), Ty^*_j(t), Tz^*_j(t)$. We propose the functions $Tx^*_j(t), Ty^*_j(t)$ and $Tz^*_j(t)$ be of the polynomial type:

$$\begin{aligned} Tx^*_j(t) &= \alpha^x_0 + \alpha^x_1 t + \alpha^x_2 t^2 + \ldots + \alpha^x_{2n} t^{2n}, \quad j=0,\ldots,k \\ Ty^*_j(t) &= \alpha^y_0 + \alpha^y_1 t + \alpha^y_2 t^2 + \ldots + \alpha^y_{2n} t^{2n}, \quad j=0,\ldots,l \\ Tz^*_j(t) &= \alpha^z_0 + \alpha^z_1 t + \alpha^z_2 t^2 + \ldots + \alpha^z_{2n} t^{2n}, \quad j=0,\ldots,m \end{aligned} \tag{9}$$

Developing the subindex j, the equations (7), take the form of (8):

$$\begin{aligned} Tx^*_k(t) &= \zeta^x_0 + \zeta^x_1 t + \zeta^x_2 t^2 + \ldots + \zeta^x_{2n} t^{2n}, \\ Ty^*_l(t) &= \zeta^y_0 + \zeta^y_1 t + \zeta^y_2 t^2 + \ldots + \zeta^y_{2n} t^{2n}, \\ Tz^*_m(t) &= \zeta^z_0 + \zeta^z_1 t + \zeta^z_2 t^2 + \ldots + \zeta^z_{2n} t^{2n} \end{aligned} \tag{10}$$

Denoting that the coefficients proposed should be different for each function $Tx^*_j(t)$, $Ty^*_j(t)$ and $Tz^*_j(t)$.

Estimate of the coefficients $\alpha_i^x, \alpha_i^y, \alpha_i^z$, $\beta_i^x, \beta_i^y, \beta_i^z$,..., $\zeta_i^x, \zeta_i^y, \zeta_i^z$, *i*=0,…,*n*+2. A polynomial type of equations system are proposed to calculate the coefficients of each function $Tx_j^*(t)$, $Ty_j^*(t)$ and $Tz_j^*(t)$ of (9). The values: a_x, a_y and a_z represent the initial position of the trajectory *TFP*, they are included in the system of equations (9), (10) and (11).

To obtain the function $Tx_j^*(t)$ with *j*=0, we have:

$$\begin{aligned} &\alpha_0^x + \alpha_1^x t_1 + \alpha_2^x t_1^2 + \ldots + \alpha_{2n}^x t_1^{2n} = ax \\ &\alpha_0^x + \alpha_1^x t_{2n} + \ldots + \alpha_{2n}^x t_{2n}^{2n} = Tx_{n0}(t_{2n}) \\ &\alpha_0^x + \alpha_1^x t_{2n+1} + \ldots + \alpha_{2n}^x t_{2n+1}^{2n} = Tx_{n0}(t_{2n+1}) \end{aligned} \tag{11}$$

To obtain the function $Ty_j^*(t)$ with *j*=0, we have:

$$\begin{aligned} &\alpha_0^y + \alpha_1^y t_1 + \alpha_2^y t_1^2 + \ldots + \alpha_{2n}^y t_1^{2n} = ay \\ &\alpha_0^y + \alpha_1^y t_{2n} + \ldots + \alpha_{2n}^y t_{2n}^{2n} = Ty_{n0}(t_{2n}) \\ &\alpha_0^y + \alpha_1^y t_{2n+1} + \ldots + \alpha_{2n}^y t_{2n+1}^{2n} = Ty_{n0}(t_{2n+1}) \end{aligned} \tag{12}$$

And to obtain $Tz_j^*(t)$ with *j*=0, we have:

$$\begin{aligned} &\alpha_0^z + \alpha_1^z t_1 + \alpha_2^z t_1^2 + \ldots + \alpha_{2n}^z t_1^{2n} = az \\ &\alpha_0^z + \alpha_1^z t_{2n} + \ldots + \alpha_{2n}^z t_{2n}^{2n} = Tz_{n0}(t_{2n}) \\ &\alpha_0^z + \alpha_1^z t_{2n+1} + \ldots + \alpha_{2n}^z t_{2n+1}^{2n} = Tz_{n0}(t_{2n+1}) \end{aligned} \tag{13}$$

An Example of Application. Two objects are moving in parabolic trajectory in the bi-dimensional space as shown in fig. 2 with the following data: *TFL_1*, Initial position: X_{01}=0.0m, Y_{01} =5.0m, Z_{10} =0.0m, V_{x01} =8.0m/s, V_{y01} =40.0m/s, Interception time 1: t_2 =6.0s, Interval of time Δt_2 =0.01s, time to catch t_3 =10.1s. *TFL_2*, Initial position: X_{02}=0.0m, Y_{02} =10.0m, Z_{20} =0.0m, V_{x02} =6.5m/s, V_{y02} =6.5m/s, Interception time 2. The delay time is: t_1=0.5s.the acceleration terms are: $a_{x1}=a_{x2}=a_{z1}=a_{z2}=0$ and $a_{y1}=a_{y2}$= g = 9.81 m/s, the initial position of the following-trajectory: a_x=20m, a_y=30m. [8-9]. To design the following trajectory *TFP* to intercept the two objects using the defined intervals of time to avoid the impact. The integration step is Δt =0.1 s. The delayed time: t_1 =0.5s. The initial position of *TFP* is ($a_x=r_x, a_y=r_y$,0). Using the proposed methodology above described.

Solution of the example: Applying the steps of the methodology of multiple interception, we have:

The trajectories *TFLi*, *i*=1,2. The parabolic trajectories can be mathematically described as follows:

$$TFL_1 \begin{cases} X_1(t) = x_{01} + V_{x01}t + \frac{1}{2}a_{x1}t \\ Y_1(t) = y_{01} + V_{y01}t + \frac{1}{2}a_{y1}t \\ Z_1(t) = 0 \end{cases} \tag{14}$$

$$TFL_2 \begin{cases} X_2(t) = x_{02} + V_{x02}t + \frac{1}{2}a_{x2}t \\ Y_2(t) = y_{02} + V_{y02}t + \frac{1}{2}a_{y2}t \\ Z_2(t) = 0 \end{cases} \tag{15}$$

The function *TFP*. The form of the function *TFP*, include the initial position of *TFL_1* and *TFL_2*. To calculate the term A_1^* of the term of the component X(t), see (16) a equation system of 9x9, is proposed to find the unknown variables: $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8$. The number of incognits most be equal to the number of the terms of the *TFLi* function.

Table 2. Forming the equation system to obtain A_1*

equation	A_1	B_1	C_1
$f_1 = \propto_0 + \sum_{i=0}^{n=8} \propto_i t_1 =$	r_x		
$\dot{f}_1 \quad =$	0		
$\ddot{f}_1 \quad =$	0		
$f_2 = \propto_0 + \sum_{i=0}^{n=8} \propto_i t_2 =$	x_{01}		
$\dot{f}_2 \quad =$	0		
$\ddot{f}_2 \quad =$	0		
$f_3 = \propto_0 + \sum_{i=0}^{n=8} \propto_i t_3 =$	x_{02}		
$\dot{f}_3 \quad =$	0		
$\ddot{f}_3 \quad =$	0		

Solving the system equation mentioned above, we obtain the unknown variables: The component A_1^* of the follower-trajectory: *TFP* are:

$$\begin{aligned} &A_1^* = \\ &\alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \alpha_4 t^4 + \alpha_5 t^5 + \alpha_6 t^6 + \alpha_7 t^7 + \alpha_8 t^8 \\ &\dot{A}_1^* = \\ &\alpha_1 + 2\alpha_2 t + 3\alpha_3 t^2 + 4\alpha_4 t^3 + 5\alpha_5 t^4 + 6\alpha_6 t^5 + 7\alpha_7 t^6 + 8\alpha_8 t^7 \\ &\boldsymbol{\ddot{A}_1^*} = \\ &\boldsymbol{2\alpha_2 + 6\alpha_3 t + 12\alpha_4 t^2 + 20\alpha_5 t^3 + 30\alpha_6 t^4 + 42\alpha_7 t^5 + 56\alpha_8 t^6} \end{aligned} \tag{16}$$

A (t)

```
f1 = α0 + α1 t1 + α2 t1^2 + α3 t1^3 + α4 t1^4 + α5 t1^5 + α6 t1^6 + α7 t1^7 + α8 t1^8 == rx
f2 = 0 + α1 + 2 α2 t1 + 3 α3 t1^2 + 4 α4 t1^3 + 5 α5 t1^4 + 6 α6 t1^5 + 7 α7 t1^6 + 8 α8 t1^7 == 0
f3 = 0 + 0 + 2 α2 + 6 α3 t1 + 12 α4 t1^2 + 20 α5 t1^3 + 30 α6 t1^4 + 42 α7 t1^5 + 56 α8 t1^6 == 0

f4 = α0 + α1 t2 + α2 t2^2 + α3 t2^3 + α4 t2^4 + α5 t2^5 + α6 t2^6 + α7 t2^7 + α8 t2^8 == x01
f5 = 0 + α1 + 2 α2 t2 + 3 α3 t2^2 + 4 α4 t2^3 + 5 α5 t2^4 + 6 α6 t2^5 + 7 α7 t2^6 + 8 α8 t2^7 == 0
f6 = 0 + 0 + 2 α2 + 6 α3 t2 + 12 α4 t2^2 + 20 α5 t2^3 + 30 α6 t2^4 + 42 α7 t2^5 + 56 α8 t2^6 == 0

f7 = α0 + α1 t3 + α2 t3^2 + α3 t3^3 + α4 t3^4 + α5 t3^5 + α6 t3^6 + α7 t3^7 + α8 t3^8 == x02
f8 = 0 + α1 + 2 α2 t3 + 3 α3 t3^2 + 4 α4 t3^3 + 5 α5 t3^4 + 6 α6 t3^5 + 7 α7 t3^6 + 8 α8 t3^7 == 0
f9 = 0 + 0 + 2 α2 + 6 α3 t3 + 12 α4 t3^2 + 20 α5 t3^3 + 30 α6 t3^4 + 42 α7 t3^5 + 56 α8 t3^6 == 0
Sol = Solve[{f1, f2, f3, f4, f5, f6, f7, f8, f9}, {α0, α1, α2, α3, α4, α5, α6, α7, α8}];
s = {α0, α1, α2, α3, α4, α5, α6, α7, α8} /. Sol;
α0 = s[[1]][[1]];
α1 = s[[1]][[2]];
α2 = s[[1]][[3]];
α3 = s[[1]][[4]];
α4 = s[[1]][[5]];
α5 = s[[1]][[6]];
α6 = s[[1]][[7]];
α7 = s[[1]][[8]];
α8 = s[[1]][[9]];

A1 = FullSimplify[α0 + α1 t + α2 t^2 + α3 t^3 + α4 t^4 + α5 t^5 + α6 t^6 + α7 t^7 + α8 t^8]
A1P = FullSimplify[α1 + 2 α2 t + 3 α3 t^2 + 4 α4 t^3 + 5 α5 t^4 + 6 α6 t^5 + 7 α7 t^6 + 8 α8 t^7]
A1PP = FullSimplify[2 α2 + 6 α3 t + 12 α4 t^2 + 20 α5 t^3 + 30 α6 t^4 + 42 α7 t^5 + 56 α8 t^6]
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Fig.4 Programming code to solve the equation system, using the mathematica® 5.0 software.

Then, we have the polynomials components:

$$TPL = \begin{cases} X^*(t) = \boxed{A_1^*} + B_1^* + C_1^* \\ Y^*(t) = D_1^* + E_1^* + F_1^* \\ Z^*(t) = G_1^* + H_1^* + I_1^* \end{cases}$$
$$T\dot{P}L = \begin{cases} \dot{X}^*(t) = \boxed{\dot{A}_1^*} + \dot{B}_1^* + \dot{C}_1^* \\ \dot{Y}^*(t) = \dot{D}_1^* + \dot{E}_1^* + \dot{F}_1^* \\ \dot{Z}^*(t) = \dot{G}_1^* + \dot{H}_1^* + \dot{I}_1^* \end{cases} \qquad (17)$$
$$T\ddot{P}L = \begin{cases} \ddot{X}^*(t) = \boxed{\ddot{A}_1^*} + \ddot{B}_1^* + \ddot{C}_1^* \\ \ddot{Y}^*(t) = \ddot{D}_1^* + \ddot{E}_1^* + \ddot{F}_1^* \\ \ddot{Z}^*(t) = \ddot{G}_1^* + \ddot{H}_1^* + \ddot{I}_1^* \end{cases}$$

Using the similar technique above mentionated for each other terms of the functions, see (17).

Numerical Results: Finally, substituting the coefficients values in (18) and adding in (17), we obtain the components of the trajectory *TFP*, wich is shown in fig.3 and 4.

$$\begin{aligned} X^*(t) &= 19.5 + 3.6t - 8.7t^2 - 8.9t^3 - 3.7t^4 + 0.8t^5 \\ &\quad - 0.09t^6 + 0.005t^7 - 0.00013t^8 \\ Y^*(t) &= 31.5 - 10.6t + 27.02t^2 - 31.1t^3 + 16.1t^4 \\ &\quad - 4.0t^5 + 0.5t^6 - 0.03t^7 \\ &\quad - 0.0008t^8 \end{aligned}$$

$$\begin{aligned} \dot{X}^*(t) &= 3.66 - 17.5t + 26.81t^2 - 14.92t^3 + 4.03t^4 \\ &\quad - 0.57t^5 + 0.04t^6 - 0.001t^7 \\ \dot{Y}^*(t) &= -10.6 + 54.0t - 93.3t^2 + 64.6t^3 - 20.1t^4 \\ &\quad + 3.08t^5 - 0.23t^6 + 0.006t^7 \end{aligned}$$

$$\begin{aligned} \ddot{X}^*(t) &= -17.46 + 53.62t - 44.76t^2 + 16.13t^3 \\ &\quad - 2.83t^4 + 0.23t^5 - 0.008t^6 \\ \ddot{Y}^*(t) &= -6.26 - 6.61t + 53.32t^2 - 33.89t^3 \\ &\quad + 7.83t^4 - 0.77t^5 + 0.03t^6 \end{aligned} \qquad (18)$$

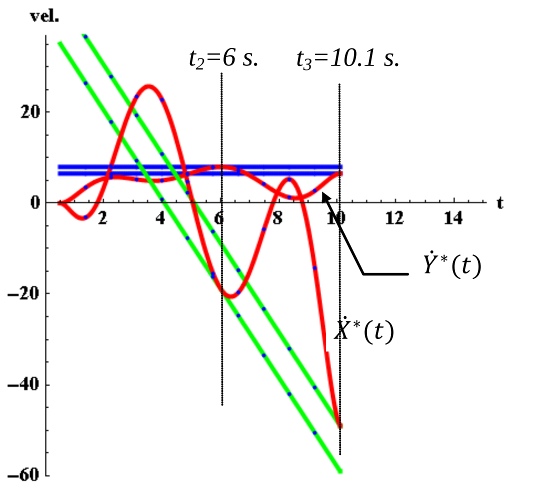

Fig. 5. Space-time phenomenon interception. Velocity.

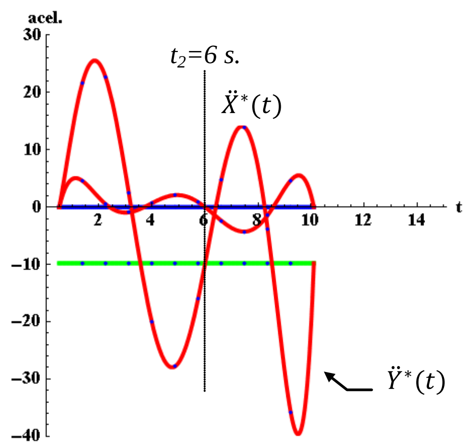

Fig. 6. Space-time phenomenon interception. Acceleration.

The Fig. 5, 6 and 7, shows the x-y-t graphics of the two TFL_1 and TFL_2 trajectories and the follower-trajectory *TFP*. The speed of the trajectory *TFP* is equal with respect to the trajectories TFL_1 y TFL_2 in the intervals of time Δt_2, Δt_4, also the direction of the trajectory *TFP* attempts to equal the direction of the *TFL´s* in the intervals of time *Δt´s*.

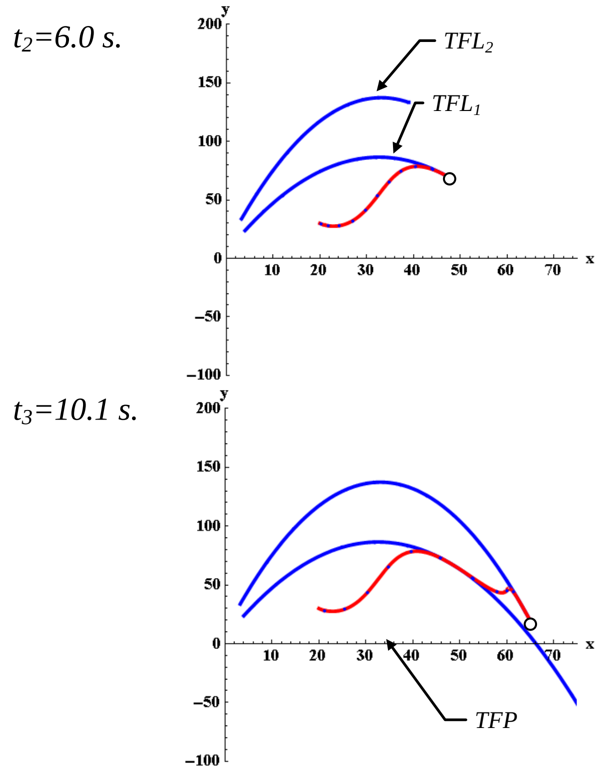

Fig. 7. The physical phenomenon of interception. Position.

Conclusions

This paper presented a novel approach to on-line, robot-motion planning for moving-objects multiple 3D object interception using a polynomial interpolation approach.

In the intervals of time the follower trajectory attempts to equal the direction of the trajectories *TFL´s*, allowing the robot captures the objects without impact. The implementation of the proposed methodology has been illustrated via simulation examples; here only one of them was presented. It has been clearly shown that the methodology proposed herein yields results favorable over the conventional tracking one object methods.

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Currículo corto de los autores

Juan Alejandro Flores Campos received the B.Eng. degree in Mechanical Electrical Engineering and the M.Sc degree in Mechanical Engineering from National Autonomous University of México, (UNAM), Mexico City, México in 1995 and 1998, respectively, and the Ph.D degree in Mechanical Engineering from National Polytechnic Institute (IPN) in 2005, where he is currently a researcher and professor of the mecatronics department.

Emmanuel Carlos Dean León received a B. Eng. degree on Industrial Robotics from the Escuela Superior de Ingeniería Mecánica y Eléctrica (ESIME-IPN), Mexico, in 2000; his M.Sc. degree in Electrical Engineering from the Research Centre for Advanced Studies (CINVESTAV), Mexico, in 2003; and his Ph.D in Electrical Engineering, from the Research Centre for Advanced Studies (CINVESTAV), Mexico, in 2006. He is regular member of the National Researcher System. His research interests include collaborative multirobots, control theory, visual servoing, computer vision and robotics.

Cándido Palacios Montúfar He gained his Bachelor degree at the Mechanical Engineering Institute of the University of Friendly People in Moscow, URSS, and his Master in Science in Mechanical Engineering as well at the National Polytechnic Institute. During 1967-1980 he was invited by the University of Newcastle Upon Tyne as Visiting Fellow, where along with other colleagues he formed a research team to carry out and develop research of kinematics and dynamics manipulators.