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Fail-Safe Motion Planning of Autonomous Vehicles

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Abstract— **Formally verified methods for motion planning are required in order to guarantee safety for autonomous vehicles. In particular, we consider trajectory generation by considering the most probable trajectory of other traffic participants. However, if the surrounding vehicles perform unexpected maneuvers, a collision might be inevitable. In this paper, a fail-safe motion planner is developed, which generates optimal trajectories, yet guarantees safety at all times. Safety is achieved by maintaining an emergency maneuver which can safely bring the host vehicle to a stop while avoiding any collision. The emergency maneuver is computed by considering for a given time horizon the occupancy prediction which encloses all possible trajectories of the other traffic participants. The performance of the approach is evaluated through simulation against real traffic data.**

I. Introduction

Autonomous vehicles are expected to become the most viable means of transportation by 2040, accounting for more than 75% of the cars on the roadway[1]. Increased safety, traffic flow, and comfort are among the most prominent advantages introduced by automated driving. Since a fully autonomous vehicle relieves humans of all driving duties, the safe operation of vehicles in a dynamic environment (i.e. the position of obstacles changes) must be guaranteed.

A key issue in guaranteeing the safety of autonomous vehicles is the generation of trajectories based on possible trajectories of other traffic participants. However, relying only on the most probable trajectory of the other traffic participants is not safe, since unexpected maneuvers might result in inevitable collisions. Fail-safe motion planning is therefore required in order to safely react in any traffic scenario. The main idea is the following: First, a trajectory is generated considering the most probable trajectory of the other vehicles. Then, an emergency maneuver which accounts for every possible action of the other traffic participants is kept available at each time step. If no other further feasible trajectory is found, then the emergency maneuver is applied until either the host vehicle is brought to standstill, or a new feasible trajectory is found.

Different methods are already available for motion planning and can be categorized as follows: 1) planning in discrete space (e.g.: grid-based approaches [15], planning using motion primitives [1], rapidly-exploring random trees [2]–[4], and road maps [5]–[9]); 2) planning in continuous space (e.g. optimal control, model predictive control [10]–[13], and elastic bands [14]). A survey on existing algorithms for collision-free trajectory planning for mobile robots can be found in [16].

1) Planning in discrete space: Sampling-based motion planning algorithms such as Probabilistic Road Maps (PRM) [5]–[9] or Rapidly-Exploring Random Trees (RRT) [2]–[4] demonstrate good performance in practice, especially for path planning in high-dimensional non-convex state spaces. However, the performance of sampling-based approaches depends on control inputs used to explore the configuration space. To decrease the computational cost required by sampling, predefined and parametrized trajectories which are often referred to as a motion primitives [1] (e.g. turn left, right turn, go straight, etc) can be used. The maneuvers are constructed such that they can be smoothly connected according to a maneuver automaton [17]. Since the motion primitives are computed off-line, the proposed planning algorithms are suitable for real-time applications. Construction of formally verified maneuver automata using reachability analysis is investigated in [18]. The authors of [19] use a heuristic graph search to find a feasible path. In [20], the motion primitives are modeled as a hybrid system, where the discrete states are the predefined trajectories and the control input used to transition from one state to another is defined by maneuvers. Thus, the optimal path is found by solving a classic hybrid optimal control problem [21].

2) Planning in continuous space: To generate a trajectory directly in continuous space, elastic bands have been introduced [14]. Elastic bands are paths which can be deformed to react to changes in the environment. This approach is used for various purposes, such as emergency maneuver generation [22], trajectory planning [23], or adaptive cruise control [24], where only one path is computed. Since a single elastic band might fail to describe a desired path, several elastic bands are generated in [25]. A single solution is then selected based on a given cost function.

For optimal trajectories which consider constraints, optimal control or MPC is used [10]–[13]. In [10], MPC is utilized for trajectory planning to prevent lane departure. Collision-free trajectories are developed in [11], which takes static obstacles into account. In [12], collision avoidance is achieved through steering and braking, and it is assumed that the obstacle moves with constant velocity.

Guaranteeing safety and comfort for motion planning in a dynamic environment is a major issue due to uncertainties introduced by the infinite number of possible trajectories of other traffic participants. However, most of the time, comfort and safety are opposite requirements, e.g. in an emergency situation, a jerky maneuver might be useful in avoiding a collision. While much work already exists both

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[1] http://www.ieee.org/about/news/2012/5september_2_2012.html

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on emergency maneuver and optimal trajectory generation, motion planning which simultaneously considers safety and comfort has not yet been addressed. Some attempts toward safe motion planning already exist. The authors of [26] employ emergency maneuvers at the end of the computed trajectory, but they do not consider all possible trajectories of the surrounding vehicles. Instead, viable inter-vehicle communication and future trajectories are assumed to be known [26].

The main contribution of this paper is the development of a fail-safe motion planner for autonomous vehicles, which simultaneously guarantees safety and comfort by combining an optimal trajectory with an emergency maneuver. Unlike previous work, our framework generates two trajectories: First, an optimal path considering the most likely movement of leading vehicles is computed for a given time horizon. Then, for each time step an emergency maneuver is kept available which takes the overapproximative future occupancy of surrounding traffic participants into account.

The remainder of the paper is structured as follows: In Sec. II, an overview of the approach is presented, including the vehicle system dynamic model and the imposed constraints. The fail-safe motion planner is described in detail in Sec. III. This algorithm is evaluated against real traffic data on a highway in Sec. IV. Finally, the conclusions are briefly presented in Sec. V.

II. Overview

Consider a road network, specified by adjacent lanes with arbitrary curvature; a possibility for efficiently representing road networks can be found in [27]. In the following, we consider that $lane_i$ is a drivable path determined by its left and right borders, $b_{\mathrm{L,i}}$ and $b_{\mathrm{R,i}}$, each defined as a polyline, see Fig. 1. We assume that the adjacency between lanes is known. For simplicity, *lanes* will refer to the union of all lanes which can be reached by the host vehicle in a given time interval: $lanes = \bigcup_i lane_i$.

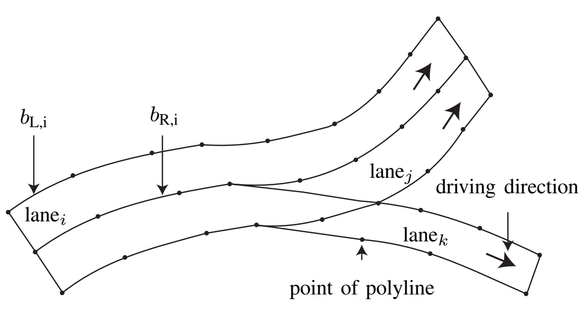


Fig. 1: Road network.

Each vehicle is uniquely described by its position (s_x, s_y) within the lanes' borders, by considering a global coordinate system. The *host vehicle* is defined as the vehicle which is controlled using our approach. The other traffic participants positioned ahead represent the *leading vehicles*. In order to focus on the novel aspects in this paper, we intentionally only consider a single leading vehicle. In principle however, this approach works for arbitrarily many leading vehicles.

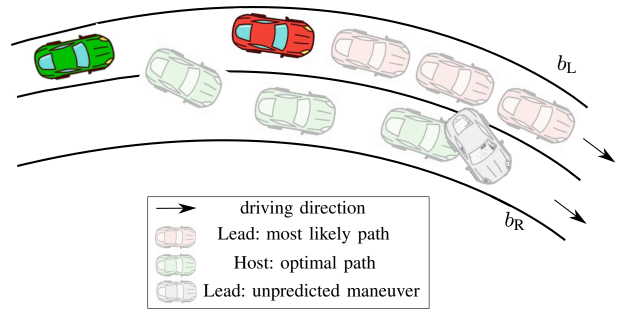


Fig. 2: A trajectory for the host vehicle is generated considering the most likely path of the leading vehicle. However, if the leading vehicle performs an emergency maneuver, the generated path for the host vehicle is no longer safe.

The problem addressed in this paper is the design of a motion planner for the host vehicle, such that the generated trajectory is smooth and any possible collision with other traffic participants is avoided. Next, let us define the system dynamics of the host vehicle.

A. Modeling

We model the vehicle as in [12], where the notation $\dot{x}$ represents the first derivative of x:

$$\begin{aligned} \dot{s}_x &= v\cos\theta, \\ \dot{s}_y &= v\sin\theta, \\ \dot{\theta} &= \frac{v\ \delta}{l\left[1+\left[\dfrac{v}{v_{\mathrm{ch}}}\right]^2\right]}, \\ \dot{\delta} &= u_1, \\ \dot{v} &= u_2, \end{aligned} \tag{1}$$

where the position s_x, s_y, velocity v, yaw angle θ, and steering angle δ represent the state variables. The steering rate u_1 and the acceleration u_2 are the control inputs. In addition, two parameters are used: the car length l and velocity v_{ch}, which depends on mass and cornering stiffness, and characterizes the steady-state dynamics of the bicycle model. The notation $\dot{x}$ represents the first derivative of x. The following constraints on state and inputs must be satisfied:

$$0 \le v \le v_{\mathrm{max}}, \tag{2}$$

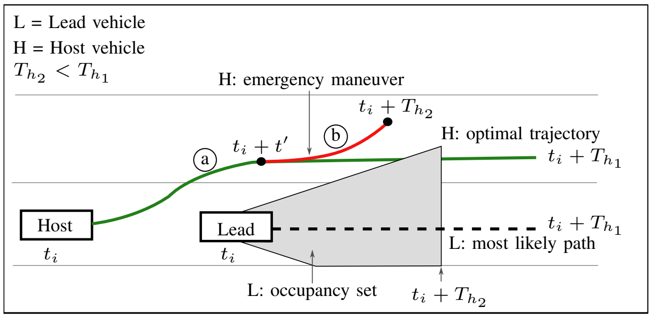
$$a_{min} \le u_2 \le a_{max}. \tag{3}$$

$$(s_x, s_y) \in lanes, \tag{4}$$

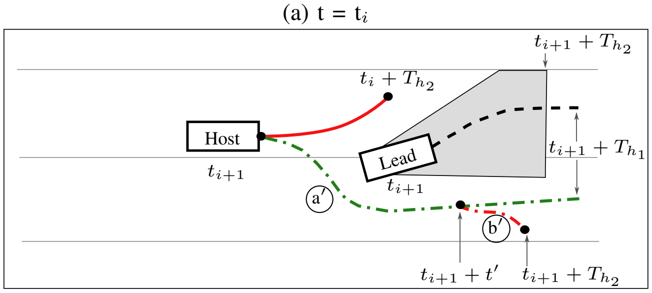
$$\delta_{\mathrm{min}} \le \delta \le \delta_{\mathrm{max}}, \tag{5}$$

$$\dot{\delta}_{\mathrm{min}} \le u_1 \le \dot{\delta}_{\mathrm{max}}. \tag{6}$$

While the first two inequalities (2)-(3) refer to physical constraints, i.e. the possible values of the velocity and acceleration, the last three constraints refer to safety (4) and comfort (5)-(6), such that the vehicle does not exit the



(a) t = t$_i$



(b) t = t$_{i+1}$

Fig. 3: At each time, an emergency maneuver which accounts for every possible maneuver of the leading vehicle is kept available. If there is no other further trajectory available, the emergency maneuver is applied, which can safely bring the host vehicle to a standstill.

lanes' boundaries and the generated trajectory is smooth. The parameters for the velocity $v_{\max}$, the acceleration $a_{\min}$, $a_{\max}$, and the steering angle $\delta_{\min}$, $\delta_{\max}$ are given.

III. Fail-safe Motion Planner

The main idea is to design a three-step motion planner which accounts for changes in the environment and at the same time maintains comfortable motions. First, the most probable maneuver of the surrounding vehicle[2] is computed. Then, an optimal trajectory of the host vehicle is generated for a given time horizon T_{h_1}, so that no collision occurs according to the assumed behavior of the other vehicle, as illustrated in Fig. 3. In the second step, an emergency maneuver is generated which can bring the host vehicle to a standstill. To guarantee safety, all possible trajectories of the leading vehicle must be considered. To this end, an overapproximative occupancy set which encloses all possible occupancies is computed [28] for a given time horizon T_{h_2}.

Let us denote the segment of the optimal trajectory computed for a given time interval $[t, t+t']$ as ⓐ, see Fig. 3a. Using optimal control, we generate a collision-free emergency maneuver ⓑ, such that the fail-safe trajectory determined by concatenation of ⓐ and ⓑ does not intersect with the occupancy set of the lead vehicle, for any intermediate time interval up to the time horizon T_{h_2}. Thus, for a given time horizon, no matter what the trajectory of the lead vehicle is, there exists an emergency maneuver which can safely bring the host vehicle to standstill, as depicted in Fig. 3a. In the third step, after new measurements are received (see Fig. 3b), it is evaluated whether the fail safe trajectory should be executed or the optimal trajectory should be continued. If there exists any further fail-safe maneuver obtained by connecting an optimal trajectory (a') with an emergency maneuver (b'), which does not intersect with the new computed occupancy set, then the optimal trajectory (a') is followed. If several feasible trajectories are found, the optimal one is chosen. Otherwise, if there exists no other collision-free trajectory, then the previously computed emergency maneuver ⓑ is applied. The general architecture is presented in Fig. 4. In the following subsections we describe the generation of the optimal and emergency trajectory in more detail.

[2]We consider as surrounding vehicles the other traffic participants which are situated in front of the host vehicle, in the same lane, or on the adjacent lanes, whose x-position (considered in the driving direction) is similar to the host vehicle. Since each vehicle is responsible for not colliding with the other vehicles, and driving backwards is prohibited on the highways, the vehicles positioned behind the host vehicle are not considered.

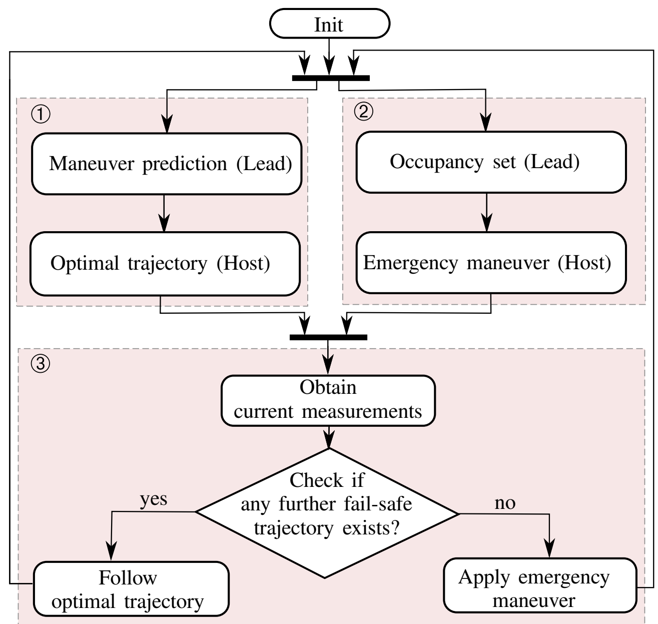


Fig. 4: General architecture of the proposed approach.

① *Optimal trajectory*

To generate an optimal trajectory, the most likely maneuver of the leading vehicle must first be computed. Different approaches for computing the most probable trajectory of the leading vehicle already exist: by assuming constant yaw rate and acceleration (CYRA) [29] or by using a maneuver recognition module (MRM) [30]. Since MRM shows higher accuracy compared with CYRA for a longer time horizon prediction [30], the MRM approach is used in the following to generate the most probable trajectory of the leading vehicle.

The main idea of the MRM is to generate a trajectory prediction based on the detection of the goal lane, i.e. the lane towards which the leading vehicle is driving. Three basic maneuvers are considered: keep lane, change lane, and turn;

obviously, the other possible maneuvers can be seen as a combination of those basic maneuvers. It is assumed that for any maneuver execution, the target position of a vehicle is on the center-line of a lane. To compute the most likely trajectory of the lead vehicle, a comparison between the current path of the vehicle and the center-line of a given lane is performed. The prediction of the most likely trajectory is computed for each time t_i within a given time horizon T_{h_1}. For more details, the reader is referred to [30]. Moreover, for each time t_i, a polygon which represents the car occupancy is associated with each position of the computed trajectory. The predicted polygons of the lead vehicle are then embedded as constraints in the trajectory generation problem of the host vehicle, as later explained in more detail.

After the most probable path of the leading vehicle is computed, an optimal trajectory of the host vehicle is generated for the same time horizon T_{h_1}. Generating trajectories that utilize optimal control or MPC, which must satisfy a set of given constraints, is already a mature research field. In this paper, an approach inspired by [12] is used to generate the trajectory of the host vehicle. Other approaches for computing an optimal trajectory can be used as well.

In [12], the main objective is collision avoidance through velocity reduction. Here, our aim is to generate a smooth trajectory by avoiding high jerk values. Hence, the cost function from [12] is modified to penalize deviation from the reference trajectory (which is the center-line of each lane) and to avoid the predicted polygons of the lead vehicle.

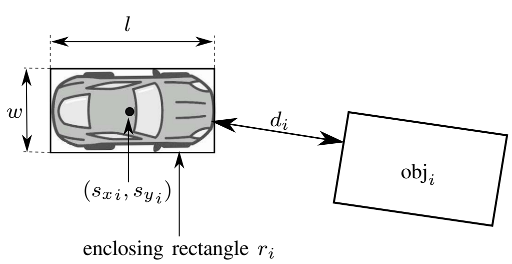


Fig. 5: Obstacle avoidance constraint.

Let us denote with $\mathrm{obj}_i, i \in \{1, \cdots, n\}$ the prediction of the leading vehicle occupancy at the i^{th} time step. In order to avoid any collision with the leading vehicle, constraints regarding the distance between the generated trajectory and the prediction obj_i must be considered for the host vehicle. Therefore, the minimum Euclidian distance d_i between the rectangle r_i, which encloses the host vehicle, and the predicted polygon obj_i, is introduced (see Fig. 5). Both polygons are computed for the i^{th} time step.

$$d_i = \min_i \ \mathit{distance}(\mathrm{r}_i, \mathrm{obj}_i). \tag{7}$$

Then, it is checked if d_i is greater or equal than a parameter λ, in order to prevent a collision with the lead vehicle. The selected cost function minimizes the variation of the velocity and the variation of the steering rate:

$$J_1 = \int_t^{t+T_{\mathrm{h}_1}} \left[\gamma_1 u_1^2 + \gamma_2 u_2^2 + \gamma_3(\theta - \theta_r)^2 + \gamma_4 \delta^2 + \gamma_5 d_r^2\right] d\tau,$$
$$\text{subject to: } (1) - (6), (s_x, s_y) \in \text{lanes}, \tag{8}$$

where θ_r is the orientation of the reference trajectory, d_r is the distance to the reference trajectory, and $\gamma_i, i \in \{1, \ldots, 5\}$ are weighting parameters. As already mentioned, the reference trajectory of the host vehicle is the center-line of the current lane; if no feasible trajectory is found using the current center-line as a reference, then the center-line of the adjacent lanes is considered to be the reference trajectory. Thus, the generated trajectory corresponds to a lane change maneuver, as depicted in Fig. 6. The collision avoidance with the surrounding vehicles is done by considering the constraints regarding the distance between the generated trajectory and the predicted trajectory of the other traffic participants, as described in (7). Of course, other approaches for trajectory generation could be used.

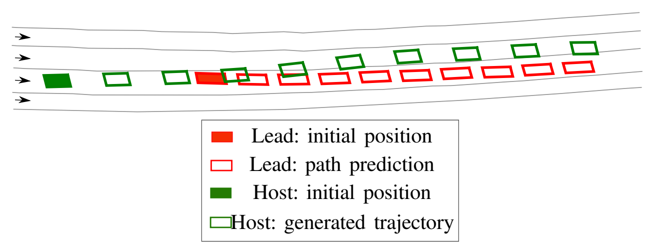


Fig. 6: Optimal trajectory generation.

② *Emergency trajectory*

The leading vehicle can perform infinitely many unpredictable maneuvers, which are not taken into account when generating an optimal trajectory, as illustrated in Fig. 2. Of course, not considering all possible maneuvers of the lead vehicle might result in a collision. Therefore, an emergency plan must be maintained, which accounts for all possible maneuvers the leading vehicle can perform in a given time horizon T_{h_2}.

In [28], a method for computing an overapproximative set which encloses all possible trajectories of the leading vehicle is proposed for a given time horizon T_{h_2}. To this end, two abstracted models for the leading vehicles are presented: They consider constraints derived from the traffic rules listed in the Vienna Convention on Road Traffic [31] and physical constraints. Next, the reachable sets for each abstraction are computed; it is proven that the intersection of reachable sets of different abstractions of other traffic participants provides the overapproximative occupancy of other traffic participants [32]. The following constraints are assumed for predicting the occupancy of the lead vehicle: driving backward and leaving the road are forbidden; the maximum absolute acceleration is limited by $a_{\max}$; longitudinal acceleration is zero when a parametrized $v_{\max}$ is reached and is inversely proportional to speed whose value is bigger than a parameter v_s, which models a maximum engine power. If one of the constraints is violated, the corresponding abstraction is no longer considered when computing the occupancy set. This approach has two main advantages: First, it guarantees that all possible trajectories are enclosed by the overapproximative occupancy set. Second, the computation time is low due to the abstracted models used for the vehicle, which makes it suitable for real-time computation. For a more detailed description, the reader is referred to [28].

As in the previous step, after the prediction of the leading vehicle is computed, a collision-free trajectory is generated for the host vehicle. The difference for computing the emergency maneuver is that the velocity must be reduced, and all possible trajectories of the lead vehicle enclosed by the entire occupancy set must be avoided. There is much research on emergency trajectory generation. However, not all possible trajectories of the leading vehicle are considered in previous work. Most work computing possible trajectories assumes that the lead vehicle is moving with constant acceleration or only considers static obstacles.

The following approach uses optimal control to generate the emergency maneuver, similar to the one used for optimal trajectory [12]. To guarantee safety, the occupancy sets are embedded in the constraint function.

The cost function is similar to the one used for optimal trajectory generation, as described in (8). The difference is that driving along a reference trajectory is no longer desired, but rather minimizing the velocity v:

$$J_2 = \int_{t}^{t+T_{h_1}} \left[\gamma_1 u_1^2 + \gamma_2 u_2^2 + \gamma_3(\theta - \theta_r)^2 + \gamma_4 \delta^2 + \gamma_5 v^2\right] d\tau$$
$$\text{subject to: } (1) - (6), (s_x, s_y) \in \text{lanes} \backslash obj_i. \tag{9}$$

IV. Numerical Experiments

Real traffic data is used to evaluate our proposed approach for fail-safe motion planning. The provided dataset is part of the Federal Highway Administration's (FHWA) Next Generation Simulation (NGSIM)[3] project, and it contains detailed vehicle trajectories. These data were collected on June 15th, 2005, on a segment of U.S. 101 Highway (Hollywood Freeway) located in Los Angeles, using eight video cameras. The vehicle trajectory data were transcribed from the taken video data using a specialized software application - the Next Generation Vehicle Interaction and Detection Environment for Operations (NG-VIDEO). For each considered vehicle, the following information is available for every 0.1 seconds: position, velocity, and acceleration.

In the simulation, the vehicles whose trajectories were recorded are considered to be leading vehicles. The host vehicle is positioned behind the leading vehicle(s); the initial velocity and acceleration are arbitrarily chosen within the given limits (see Tab. I). The parameters used for the generation of fail-safe trajectories are listed in Tab. I.

[3]http://ops.fhwa.dot.gov/trafficanalysistools/ngsim.htm

In the following, three scenarios based on the NGSIM dataset are considered: First, we present a scenario in which the motion planner does not consider fail-safe maneuvers. Then we apply our proposed algorithm to the same scenario and compare the two. Finally, multiple leading vehicles are considered.

1) Scenario 1: Here, only the predicted trajectory of the lead vehicle is taken into account when computing the motion for the host vehicle. Initially, both host and lead vehicles are situated in the same lane at a 37m distance. The initial velocity for the host and the lead vehicle are 20m/s and 13.5m/s. At time 4.5s (t_9), the lead vehicle performs an unexpected maneuver and drives towards the left lane. Since no fail-safe maneuver is considered for the host vehicle, a crash occurs, as illustrated in Fig. 7.

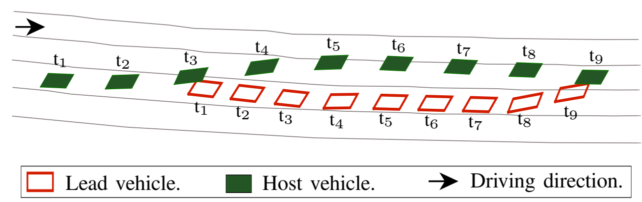


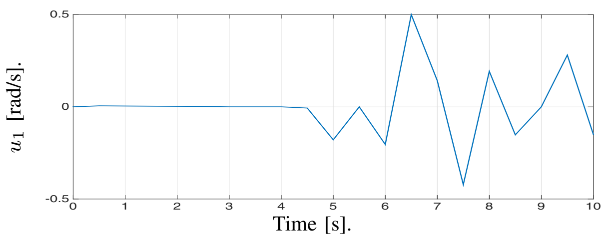
Fig. 7: Scenario 1. Simulation results.

2) Scenario 2: The same traffic scenario is considered, which differs from the host vehicle's motion planner used in the previous scenario. Here the set-based occupancy prediction is computed for the leading vehicle at each time step. Thus, all possible maneuvers are considered. At each time step, a feasible emergency maneuver is available for the host vehicle. At time 4.5s (t_9), the lead vehicle steers to the left, towards the host vehicle. This unexpected behavior triggers an emergency maneuver for the host vehicle in order to successfully avoid a collision, as depicted in Fig. 9. The inputs used to control the host vehicle are presented in Fig. 8. The control inputs u_1 and u_2 are high due to the fail-safe maneuver which is only executed in an emergency situation.

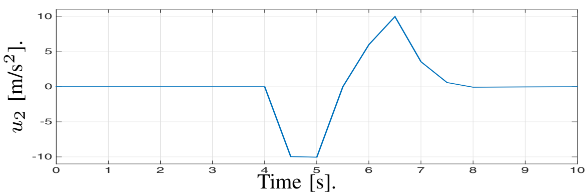
3) Scenario 3: A scenario with two surrounding vehicles is considered. The initial distances between the host vehicle and the other vehicles are 37m and 49m, and the initial velocity of the lead vehicles is 13.5m/s and 13m/s. Fig. 10 shows the measured position of the surrounding vehicles at each time step, together with the generated path of the host vehicle. The values of control inputs u_1 and u_2 are presented in Fig. 11. The lead vehicle #1 (see Fig. 10) performs an unexpected maneuver at time t_9 towards the left lane, where the host vehicle is driving. The host vehicle successfully avoids the collision by applying the available emergency maneuver. Next, at time t_{22} the lead vehicle #2 starts a change lane maneuver. At the next time step, the lane change maneuver is aborted, and the host vehicle continues driving along the planned trajectory. As it can be seen, the host vehicle's path is collision-free for the entire simulation.

TABLE I: Parameters.

Parameter [unit]	T_s [s]	T_{h_1} [s]	T_{h_2} [s]	ϵ [m]	v [m/s]	a [m/s^2]	θ [rad]	δ [rad]	w [m]	l [m]	v_{ch} [m/s]
Value/ Interval	0.5	5	1	2.4	[0, 60]	[-10, 10]	[$-\pi/2$, $\pi/2$]	[-0.5,0.5]	1.7	4	50



(a) Steering rate u_1.



(b) Acceleration u_2.

Fig. 8: Scenario 2. Control inputs.

V. Conclusions and Future Work

In this paper, a fail-safe motion planning approach for autonomous vehicles is presented. The optimality of the approach is achieved by considering the most likely trajectory of the lead vehicle. The safety of the proposed method is guaranteed by keeping an emergency maneuver available which accounts for every possible trajectory of the leading vehicle over a given time horizon. Thus, the main asset of our technique is that we are able to bring the host vehicle to a safe stop, no matter what the current maneuver of the lead vehicle is.

The approach is tested using real traffic data, and it shows that safety can indeed be achieved by considering all possible maneuvers of the leading vehicle(s). As a direction for future research, we will also consider the interaction between surrounding vehicles, such that the traffic behavior is anticipated more accurately. This would allow the host vehicle's motion planner to be smoother and more comfortable.

Acknowledgment

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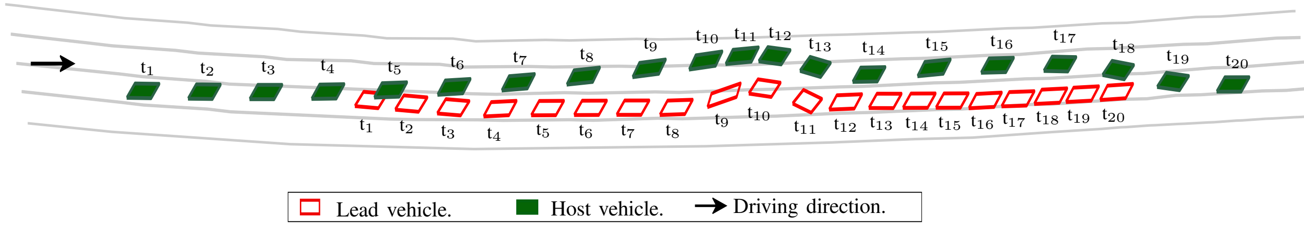


Fig. 9: Scenario 2. Simulation results.

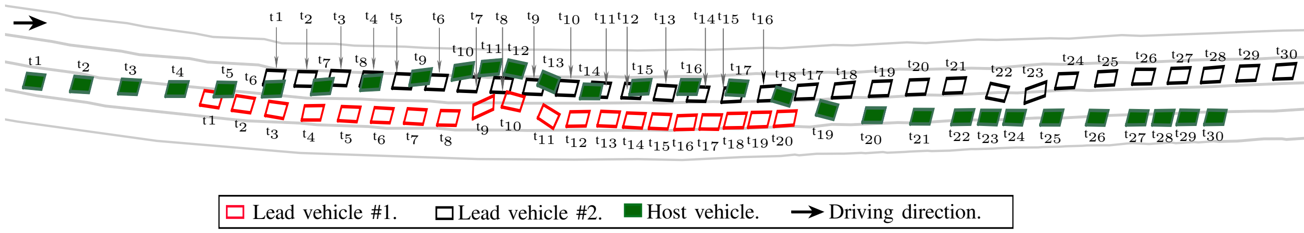
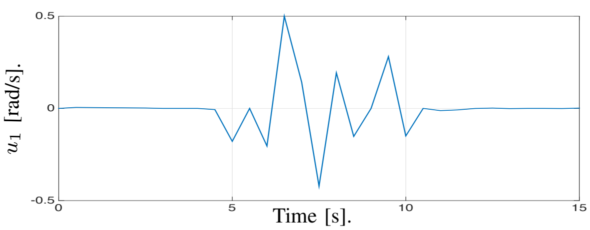
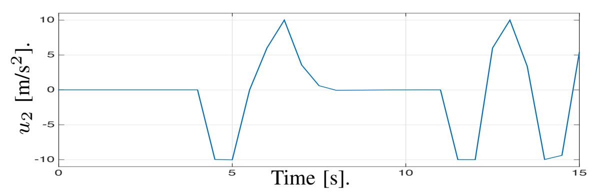


Fig. 10: Scenario 3. Simulation results.



(a) Steering rate u_1.



(b) Acceleration u_2.

Fig. 11: Scenario 3. Control inputs.

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