

Adaptive Cruise Control with Safety Guarantees for Autonomous Vehicles

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Abstract: This paper addresses the problem of following a vehicle with varying acceleration in a comfortable and safe manner. Our architecture consists of a nominal controller (here: model predictive control) and a safety controller. Although model predictive control attempts to keep a safe distance, it cannot formally guarantee it, due to the assumptions on the behavior of the leading vehicle. We address this problem by holding a formally verified safety controller available. Our novel mechanism gradually engages the safety maneuver since most critical situations resolve quickly. The overall approach is evaluated against real traffic data. The results show good position and velocity tracking performance, while safety and comfort are guaranteed.

Keywords: Autonomous vehicles, Intelligent cruise control, Safety guarantees.

1. INTRODUCTION

In recent years, one of the most important goals in the automotive industry has been to offer passengers the highest level of safety, comfort, and efficiency by partially or completely removing driving duties from humans. Studies have shown that active safety systems, such as (adaptive) cruise control, electronic stability control or lane keeping, which are already on the automotive market, can improve safety by decreasing the number of traffic accidents (Rieger et al., 2005). More specifically, Adaptive Cruise Control (ACC) as described in (ISO, 2010), can improve traffic flow and driving comfort (Ioannou et al., 1993; Ioannou and Chien, 1993); in addition to improving traffic flow and comfort, ACC systems can also reduce fuel consumption (Alam et al., 2010) and trip time (Asadi and Vadihi, 2011). An extensive survey on ACC systems can be found in (Vahidi and Eskandarian, 2003) and (Xiao and Gao, 2010).

However, most of the time, the goals of maintaining a safe distance and improving traffic flow by decreasing the inter-vehicle distance are conflicting requirements. Therefore, almost 50% of two-vehicle crashes are rear-end collisions¹. If the leading vehicle suddenly decelerates (e.g. after another vehicle cuts-in in front of the leading vehicle) and an emergency situation occurs, the ACC is deactivated and the driver becomes responsible for (fully) braking. In the near future, it is assumed that autonomous vehicles will take over the driving duties from humans completely, including safely reacting in these emergency situations. Thus, a new ACC concept which can always guarantee safety and comfort is required.

1.1 Related Work

Different space control policies have been presented in previous work to check whether a collision can be avoided

(Marzbanrad and Karimian, 2011; Swaroop et al., 1994; Santhanakrishnan and Rajamani, 2003; Yanakiev and Kanellakopoulos, 1998). A variety of speed and deceleration profiles are proposed (Németh and Gáspár, 2015; Wilson, 2001) to maintain the inter-vehicle distance. Furthermore, most of the work in this direction considers a trade-off between safety and comfort, yet does not guarantee safety.

Recent approaches to safe adaptive cruise control use correct-by-construction control software synthesis, where the system specifications are given in Linear Temporal Logic (LTL) (Nilsson et al., 2016, 2014). The designed controller which satisfies the desired behavior is based on a discrete abstraction of the system. However, the computation of the finite abstraction is expensive, and the size of the final graph (on which the controller synthesis is based) is exponential in the length of the LTL formula and in the dimension of the system.

The ACC problem can also be addressed using control barrier functions (Mehra et al., 2015; Ames et al., 2014). These functions are used to penalize the violation of the constraints that arise from ACC specifications. Therefore, the property that the value of a control barrier function approaches infinity, as points approaching the boundaries of the safe region (i.e. safe distance becomes too short), is exploited. Finding a control barrier function, however, is not a trivial task.

Game theory techniques can also be applied to autonomous vehicles in order to increase safety and traffic throughput (Lygeros et al., 1998; Tomlin et al., 2000). Each vehicle is considered an agent, and the controller design is seen as a game between the actions of each agent and the disturbances introduced by the environment. Nevertheless, this approach has exponential complexity.

The MPC framework is widely used to tackle adaptive cruise control problems by using its capability of handling multiple constraints in a receding horizon fashion

¹ <http://www.nts.gov/safety/safety-studies/Documents/SIR1501.pdf>

(Bageshwar et al., 2004; Mayne et al., 2000; Corona et al., 2006; Stanger and del Re, 2013; Li et al., 2011; Naus et al., 2010). A benchmark setup is proposed in (Corona and Schutter, 2008) to assess different model predictive control methods used for ACC. An overview on constraint MPC can be found in (Maciejowski, 2002); for a comprehensive survey on MPC with constraints, the reader is referred to (Mayne et al., 2000). In the following, we mainly focus on previous work on ACC which uses MPC, since this work is most closely related to our approach. In (Bageshwar et al., 2004), a two-mode ACC is developed using MPC, in which controllers shift between speed control (transitional operation) and distance control (steady-state operation). The optimization problem is solved subject to desired inter-vehicle distance and acceleration limitation, which are incorporated as constraints. In (Li et al., 2011), an optimal control law is applied in order to increase tracking capabilities and fuel economy. In order to keep a safe distance between vehicles, the authors use a constant time headway spacing policy. The aim of the control problem addressed in (Corona et al., 2006) is to ensure a minimum distance between two vehicles. It is assumed that at each sample time, the host vehicle receives the future reference state of the leading vehicle. However, if the leading vehicle suddenly brakes, the host vehicle might not stop within the given safe distance.

Recently, the idea of cooperative adaptive cruise control was developed (Stanger and del Re, 2013; Öncü et al., 2014). String stability, i.e. the capacity to minimize the tracking errors in the upstream direction of convoys, which is one of the most important properties of a platoon, is addressed in (Ploeg et al., 2014; Cook, 2007; Yanakiev and Kanellakopoulos, 1998). A key component in a cooperative architecture (platoon) is inter-vehicle communication, i.e. all entities within the cooperative team know the future trajectory of the others. However, if the communication is lost and one of the vehicles performs an unexpected maneuver (e.g. fully braking), a collision might be inevitable.

1.2 Contributions

Designing an ACC concept which simultaneously considers keeping a safe distance between vehicles and avoiding jerky maneuvers is not a trivial task, as good tracking capabilities can lead to frequent emergency braking. However, if the braking is too smooth, a collision might be imminent.

The main contribution of this paper is to design a control scheme which consists of a nominal controller, which is supervised by an emergency controller; together they guarantee safety and comfort at all times. The safety is achieved by computing a correct safe distance and ensuring that the inter-vehicle distance is always larger than the safe distance. First, an optimal control input is generated utilizing MPC, which minimizes the position error and the jerk of the host vehicle, guaranteeing performance and comfort. An emergency maneuver following the optimal maneuver is kept available, which is only active as long as MPC does not provide a safe solution, due to unexpected disturbances. While switching between an intelligent cruise control and emergency control is also considered in a previous work (Mayr and Bauer, 1999),

our approach also ensures limited jerk maneuvers for both nominal and emergency controllers. Additionally, string stability is guaranteed during the nominal behavior. By utilizing our framework, the system can track the computed distance while considering the worst-case scenario when the leading vehicle suddenly fully brakes. The ride comfort is guaranteed both during the nominal control, and during emergency maneuvers, which have a gradual braking policy while ensuring safety in all circumstances.

1.3 Organization

The remainder of this paper is structured as follows: The problem description and the assumptions made throughout the paper are presented in Sec. 1.4. The vehicle model in addition to safety and comfort constraints is provided in Sec. 2. In Sec. 3, the general architecture of the ACC setup is presented. Sec. 4 first analyzes different deceleration profiles and their corresponding braking distances. Then the emergency and nominal controllers are described. Numerical evaluations are presented in Sec. 5, followed by a comparison with a state-of-the-art ACC used in the automotive industry. Finally, the conclusions and future work are presented in Sec. 6.

1.4 Problem Statement

A typical ACC does not consider an emergency brake situation where the leading vehicle can suddenly fully brake. In this situation, a collision might be imminent if the inter-vehicle distance is not large enough. The goal of this paper is to design a control scheme that (i) guarantees safety for all possible scenarios, i.e. a safe distance must be kept between vehicles, and (ii) ensures comfort at all times, i.e. there are no jerky maneuvers.

No available communication between vehicles is assumed. That is, the host vehicle does not know the future velocity/acceleration profile of the leading vehicle. However, if communication between vehicles exists, the performance of the proposed framework would be even better. If there is no preceding vehicle, ACC behaves like a typical cruise control system. Note that sensor (e.g. lidars, cameras, lasers) performance analysis is beyond the work presented in this paper.

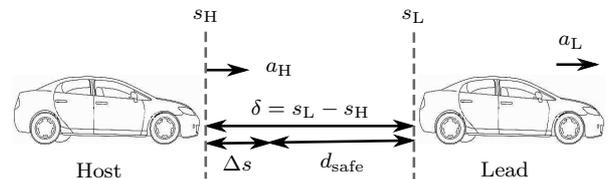


Fig. 1. Adaptive cruise control setup.

2. MODELING

In this section, we derive the mathematical model for vehicle following, as illustrated in Fig. 1. Each vehicle is described by its absolute position (s_H and s_L), velocity (v_H and v_L), and its absolute acceleration (a_H and a_L). The measured distance δ between the host and the leading vehicle is $\delta = s_L - s_H$. In the worst-case scenario, where the leading vehicle fully brakes with the minimum acceleration

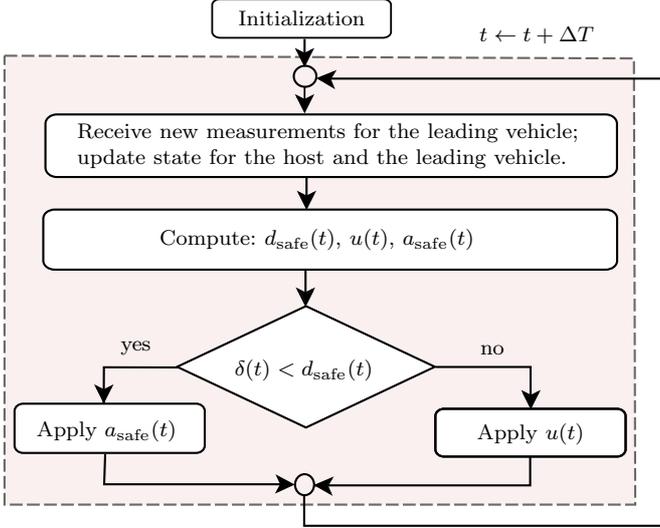


Fig. 2. Control scheme of our proposed ACC concept.

a_{\min} , the braking distance of the leading vehicle can be computed by substituting the final velocity with 0 (standstill) in the equation of motion, $0 = v_L^2 - 2|a_{\min}|d_{\text{lead}}$, so that

$$d_{\text{lead}} = \frac{v_L^2}{2|a_{\min}|}. \quad (1)$$

The model used to design the nominal controller for the ACC-equipped vehicle, considering constant velocity $a_L = 0$, is described as follows:

$$\begin{aligned} \dot{x} &= Ax + Bu, \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \\ x &= \begin{pmatrix} \Delta s \\ \Delta v \\ a_{\text{host}} \end{pmatrix}, \quad u = j_H, \\ \Delta s &= \delta - d_{\text{safe}}, \quad \Delta v = v_L - v_H, \end{aligned} \quad (2)$$

where the control variable u is the jerk of the host vehicle j_H (jerk is the time derivative of acceleration). The state and control inputs are only allowed to take values within the following intervals:

$$\begin{aligned} 0 &\leq \Delta s \leq \Delta s_{\max}, \\ a_{\min} &\leq a_H \leq a_{\max}, \\ j_{\min} &\leq j_H \leq j_{\max}, \end{aligned} \quad (3)$$

where Δs_{\max} , a_{\min} , a_{\max} , j_{\min} , and j_{\max} are user-specified parameters. An additional constraint is considered for the acceleration of the host vehicles, in order to achieve string stability (Kianfar et al., 2011):

$$a_{H,k} \leq \max_{\tau \in [k-H, k]} |a_{L,\tau}|, \quad k \in \{1, \dots, N\}, \quad (4)$$

where k is the current time instant. Parameter N is the prediction horizon, and H is the size of the time window, which must be long enough to account for delays arising in the platoon. Both N and H are hand-tuned.

3. ACC WITH SAFETY GUARANTEES

The main objective of this work is to embed standard controllers for adaptive cruise control into a framework that guarantees collision avoidance. The main idea for achieving this objective is to always hold available a safe

braking trajectory that brings the ego vehicle to a safe stop even when the preceding vehicle would suddenly fully brake. As long as the standard controller, which we refer to as the *nominal controller* from now on, provides a safe distance $\delta \geq d_{\text{safe}}$ as shown in Fig. 1, the nominal controller stays in action. Details on how to compute the safe distance d_{safe} are presented subsequently in Sec. 4.

In the event that there exists no input $u(t)$ s.t. $\delta \geq d_{\text{safe}}$, the braking trajectory is engaged, which we refer to as $a_{\text{safe}}(t)$. In our work, we do not only consider full braking, but also discuss several braking profiles with respect to the length of d_{safe} and the jerk values of the braking trajectory. A more gradual engagement of brakes decreases jerk and thus increases comfort, while enlarging the required safe distance d_{safe} . Controllers for tracking the pre-computed braking trajectory are not discussed in this work to focus on the novel aspect of guaranteeing collision avoidance. If, during the braking maneuver, the inter-vehicle distance again becomes $\delta > d_{\text{safe}}$ (since d_{safe} has shortened due to the fact that the preceding vehicle has not engaged brakes to the expected extent), the control is taken back to the nominal controller. Since (i) we choose our braking profiles such that they initially only engage mildly and (ii) in almost all cases, control quickly goes back to the nominal controller, passengers would not realize in most cases that the braking trajectory is engaged as discussed in Sec. 5.

In this work, we use model predictive control (MPC) as a nominal controller. Any controller can be used just as well as MPC in the proposed framework. MPC is used because it provides optimal solutions while attempting to meet constraints—this, however, is not always achieved in this work, due to assumptions about the behavior of the leading vehicle, which will not exactly materialize. Our MPC is computed based on the assumption that the leading vehicle moves with constant velocity, which is a reasonable assumption for optimizing ride comfort, but safety cannot be ensured since the leading vehicle might brake.

Therefore, an emergency controller has to be applied when a critical situation occurs. The control output of the MPC is denoted by $u(t)$. Our cost function for MPC is rather standard and can be formulated as a quadratic programming (QP) problem; all matrices have appropriately chosen dimensions:

$$\begin{aligned} \min_u J(x(k), u(k)) &= x_{N|k}^T P x_{N|k} + \\ &+ \sum_{i=0}^{N-1} \left(x_{i|k}^T Q x_{i|k} + u_{i|k}^T R u_{i|k} \right), \text{ subject to: (3)-(4),} \end{aligned}$$

where:

- $x_{i|k}$ and $u_{i|k}$ are the state and input at time instant i , $i \in \mathbb{N}$, $i \leq N$, based on the state measurement at time instant k ,
- $J(\cdot, \cdot)$ is the cost function,
- matrix $Q \geq 0$ is weighting the state vector,
- matrix $R > 0$ penalizes the control input,
- terminal cost P is chosen to guarantee stability.

Let us denote with $\mathcal{X}_v(\cdot)$, $v \in \{L, H\}$ the resulting position of the host vehicle H and the lead vehicle L , by applying the control input u or the acceleration a . To summarize

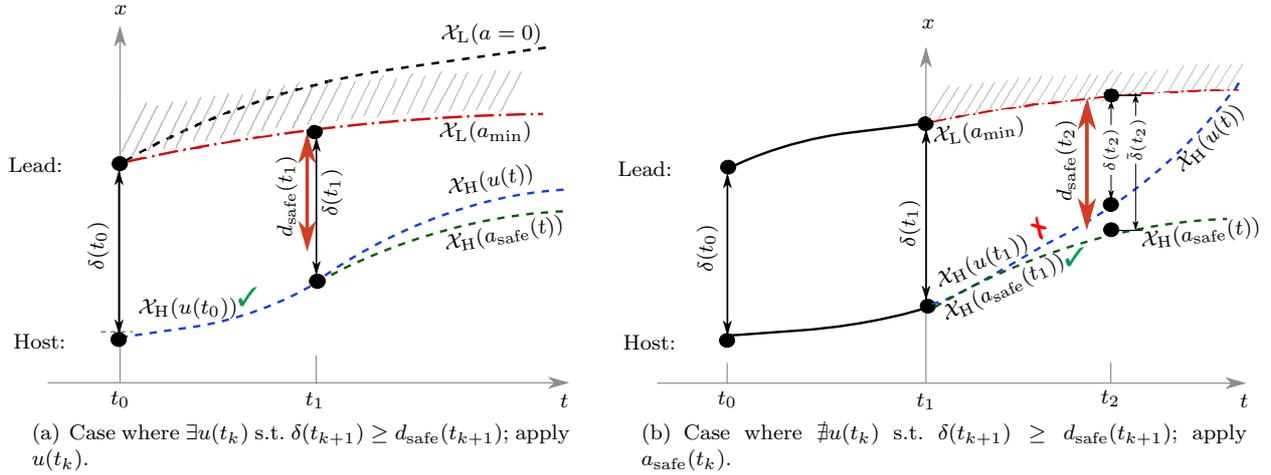


Fig. 3. Switching the control based on the current measurement.

the proposed control scheme, first, an optimal control output $u(t_k)$ is generated, under the assumption that the leading vehicle is driving with constant velocity v_L . At each sample time, we verify if by applying $u(t_k)$ for one time step, the safety distance d_{safe} is satisfied, i.e. there exists an emergency maneuver $a_{\text{safe}}(t_k)$ that can bring the host vehicle to a standstill while avoiding any collision, even in the worst case scenario when the leading vehicle brakes with full deceleration a_{min} (see Fig. 3(a)). If the verified control output $u(t_k)$ yields a safe distance, then $u(t_k)$ is applied to the system.

Let us introduce $\tilde{\delta}(t)$ as the intervehicle distance at time t , if the ego vehicle applies $a_{\text{safe}}(t)$. If no control output $u(t_k)$ is found such that the safe distance $d_{\text{safe}}(t_{k+1}) \leq \tilde{\delta}(t_{k+1})$ (as illustrated in Fig. 3(b)) is met, a gradual emergency maneuver $a_{\text{safe}}(t_k)$ is applied, which guarantees safety $\tilde{\delta}(t_{k+1}) \geq d_{\text{safe}}(t_{k+1})$, for any pre-defined deceleration profile of the leading vehicle. The emergency maneuver is applied until a new verified control output $u(t_k)$ is found.

4. BRAKING DISTANCE AND DECELERATION PROFILE

A formal analysis to compute a safe distance is presented in (Rizaldi et al., 2016), but only the case where constant acceleration is applied is considered. Here, four different deceleration profiles are analyzed. Out of those, we select the solution which guarantees safety at all times and additionally, which assures comfort, by generating minimum jerk. Based on these criteria, we propose a mixed deceleration profile $a_{\text{safe}}(t)$, and we compute the safe braking distance d_{host} , so that a collision is avoided while keeping low jerk values.

Full deceleration. The most straightforward approach is to apply constant full deceleration: $a_{\text{safe}}(t) = a_{\text{min}}$, $\forall t \geq 0$. This profile provides the smallest safe distance possible. However, applying full deceleration leads to uncomfortable driving. Moreover, due to the jerky behavior, traffic flow might not be improved with this profile.

Linear deceleration. Another possible profile is linear deceleration: $a_{\text{safe}}(t) = \frac{a_{\text{min}}t}{c}$, $\forall t \geq 0$, $c \geq \frac{v_H}{a_{\text{min}}}$,

$a_{\text{min}} \leq a_{\text{safe}}(t) < 0$. The jerk value is low since the acceleration is linearly decreasing. However, the braking distance of this profile is larger than the full deceleration profile described previously.

Exponential deceleration. The exponential deceleration profile is defined as follows:

$$a_{\text{safe}}(t) = 1 - c^t, \quad \forall t > 0, \quad c > 1, \quad a_{\text{min}} \leq a_{\text{safe}}(t) < 0. \quad (5)$$

When applying the exponential deceleration, the jerk value is even less compared to linear deceleration. Therefore, if the leading vehicle fully brakes for only one time step, the host vehicle will smoothly brake, making this deceleration profile suitable for systems whose measurements are affected by noise. In the following, the computation of the braking distance is derived: Let s be the solution of the differential equation $\dot{s} = a_{\text{safe}}(t)$. We define the braking time of the host vehicle $t_{\text{stop,H}}$ as the time when the velocity reaches 0, where the initial velocity is v_0 . The braking distance d_{host} is the exact solution of $\ddot{s} = a_{\text{safe}}(t)$, computed for the braking time $t_{\text{stop,H}}$, by double integrating the acceleration:

$$d_{\text{host}} = \frac{1}{\ln^2 c} + \frac{t_H^2}{2} - \frac{c^{t_H}}{\ln^2 c} + \frac{t_{\text{stop,H}}}{\ln c} + v_0 t_{\text{stop,H}}. \quad (6)$$

Mixed deceleration. The main drawbacks of the previous deceleration profiles are (i) if constant maximum deceleration is applied, the jerk is a Dirac function, so the value goes to infinity, and (ii) by applying linear or exponential deceleration, the jerk is comfortable, but the braking distance is too long. To overcome these disadvantages, a mixed deceleration profile is proposed. Parameter t_H represents the time when the maximum deceleration is reached during exponential deceleration so that we continue with full braking. In the following, the mixed deceleration profile is utilized.

$$a_{\text{safe}}(t) = \begin{cases} 1 - c^t & \text{if } t \leq t_H, \\ a_{\text{min}} & \text{if } t_H < t \leq t_{\text{stop,H}}, \end{cases} \quad (7)$$

$$t_H = \log_c(1 - a_{\text{min}}), \quad t > 0, \quad c > 1, \quad a_{\text{min}} \leq a_{\text{safe}}(t) < 0.$$

Let $s_L(t) = s_L(t_0) + v_L(t_0)t - \frac{1}{2}|a_{\text{min}}|t^2$ be the position and $v_L(t) = v_L(t_0) - |a_{\text{min}}|t$ the velocity of the leading

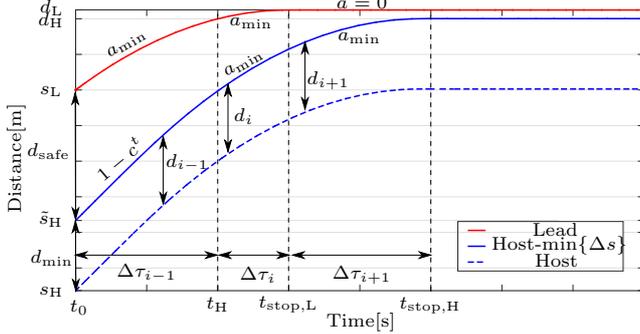


Fig. 4. Safe distance computation.

vehicle at time t , when full brake is applied; let $s_H(t)$ be the position of the host vehicle. The distance d_{safe} which guarantees safety is computed as:

$$d_{\text{safe}} = s_L(t_0) - s_H(t_0) - d_{\text{min}},$$

where the distance between $s_L(t)$ and $s_H(t)$ over a time interval $\Delta\tau_i$ is

$$d_{\text{min}} = \min_i d_i, \quad d_i = \min_{t \in \Delta\tau_i} (s_L(t) - s_H(t)).$$

To compute d_{min} , we exploit the monotonicity of d_i : Both $s_L(t)$ and $s_H(t)$ are monotonically increasing over time intervals $\Delta\tau_i = [t_{\text{min}}, t_{\text{max}}]$, $t_{\text{min}}, t_{\text{max}} \in \{t_{\text{stop,L}}, t_H, t_{\text{stop,H}}\}$, where

$$t_{\text{stop,L}} = \frac{v_L}{|a_{\text{min}}|}, \quad t_H = \frac{\ln(1 - a_{\text{min}})}{\ln(c)},$$

$$t_{\text{stop,H}} = \frac{1}{|a_{\text{min}}|} \left(\frac{1 - c^{t_H}}{\ln(c)} + v_H \right),$$

$t_{\text{stop,L}}$ is the braking time of the leading vehicle.

Next, to analyze the monotonicity of d_i , we first compute all possible permutations between $t_H, t_{\text{stop,H}}$, and $t_{\text{stop,L}}$, since the acceleration mode changes at these points in time. There are six possible scenarios, based on the applied accelerations ($a_{\text{lead}} \in \{0, a_{\text{min}}\}$ and $a_{\text{host}} \in \{0, a_{\text{min}}, 1 - c^t\}$). The cases when both vehicles are standing still and when the host vehicle is standing still and the leading vehicle is braking are not considered because they already represent safe situations; therefore, only the remaining four combinations of acceleration are analyzed, as shown in Tab. 1.

Table 1. Possible combinations of applied deceleration.

	case (a)	case (b)	case (c)	case (d)
Lead	a_{min}	standstill	standstill	a_{min}
Host	a_{min}	a_{min}	$1 - c^t$	$1 - c^t$

In the following, we compute each d_i , $i \in \{1, 2, 3, 4\}$ (for each aforementioned case) considering all possible combinations of the applied deceleration of the host and leading vehicle for each time interval $\Delta\tau_i$. For example, in Fig. 4 three different cases can be distinguished: $t \in [t_0, t_H]$: case(d), $t \in [t_H, t_{\text{stop,L}}]$: case(a); $t \in [t_{\text{stop,L}}, t_{\text{stop,H}}]$: case(b); the case(c) would occur if $t \in [t_{\text{stop,L}}, t_H]$, $t_{\text{stop,L}} \leq t_H$.

Each case is analyzed, and d_i is computed.

- **Case (a)**: $\Delta v = v_L(t) - v_H(t)$, $t \in \Delta\tau_i$;

if $\Delta v > 0$ then $\Delta s(t)$ is increasing on $\Delta\tau_i \Rightarrow$

$$d_i = s_L(t_{\text{min}}) - s_H(t_{\text{min}});$$

if $\Delta v < 0$ then $\Delta s(t)$ is decreasing on $\Delta\tau_i \Rightarrow$

$$d_i = s_L(t_{\text{max}}) - s_H(t_{\text{max}}).$$

- **Case (b)**: $\Delta v = 0 - v_H(t) < 0$ then $\Delta s(t)$ is decreasing on $\Delta\tau_i \Rightarrow d_i = s_L(t_{\text{max}}) - s_H(t_{\text{max}})$.

- **Case (c)**: $\Delta v = 0 - v_H(t) < 0$ then $\Delta s(t)$ is decreasing on $\Delta\tau_i \Rightarrow d_i = s_L(t_{\text{max}}) - s_H(t_{\text{max}})$.

- **Case (d)**: Δv is computed by integrating the corresponding acceleration difference,

$$\Delta v = a_{\text{min}}t + v_L - t - v_H - \frac{1 - c^t}{\ln(c)};$$

to find if Δs is increasing or decreasing, the solution of $\Delta v = 0$ is computed, which provides the critical point of Δs as:

$$t^* = - \frac{q \ln(c) + p \text{LambertW}\left(0, \frac{\ln(c)}{p \sqrt[3]{c^q}}\right)}{p \ln(c)},$$

where $p = (a_{\text{min}} - 1)\ln(c)$, $q = 1 + (v_L - v_H)\ln(c)$, and the *LambertW* function is the inverse function of $f(W) = We^W$. To check if $\Delta s(t)$ has a minimum or a maximum value at time t^* , we compute the second derivative of Δs , i.e. $\Delta a(t) = a_{\text{min}} - (1 - c^t)$. Since $\Delta a(t) < 0 \Rightarrow \Delta s(t^*)$ has a maximum at t^* . Therefore, the minimum of $\Delta s(t)$ can be at either t_{min} or t_{max} . Three further cases can be distinguished:

- (d.1): $t^* < t_{\text{min}} \Rightarrow \Delta s(t)$ is decreasing on $\Delta\tau \Rightarrow d_i = s_L(t_{\text{max}}) - s_H(t_{\text{max}})$;

- (d.2): $t^* > t_{\text{max}} \Rightarrow \Delta s(t)$ is increasing on $\Delta\tau \Rightarrow d_i = s_L(t_{\text{min}}) - s_H(t_{\text{min}})$;

- (d.3): $t^* \in [t_{\text{min}}, t_{\text{max}}] \Rightarrow$

$$d_i = \begin{cases} \Delta s(t_{\text{min}}), & \text{if } \Delta s(t_{\text{min}}) < \Delta s(t_{\text{max}}) \\ \Delta s(t_{\text{max}}), & \text{if } \Delta s(t_{\text{min}}) \geq \Delta s(t_{\text{max}}) \end{cases}.$$

To summarize, first the time intervals $[t_{\text{min}}, t_{\text{max}}]$, $t_{\text{min}}, t_{\text{max}} \in \{t_H, t_{\text{stop,H}}, t_{\text{stop,L}}\}$ are selected depending on the scenario. Then, d_{min} is computed accordingly, based on the applied deceleration profiles. Finally, the safe distance d_{safe} is computed such that any collision is avoided by applying the proposed deceleration profile $a_{\text{safe}}(t)$.

5. NUMERICAL EXPERIMENTS

The presented approach is evaluated with real traffic data for more than 300 vehicles. The data is collected on a segment of US highway 101 (Hollywood Freeway) located in Los Angeles, California, on June 15th, 2005, as part of the Next Generation Simulation (NGSIM)² project. In the simulations, the vehicles from the dataset are considered as leading vehicles in the ACC setup. For each vehicle, the following information is available at each sampled time: position, velocity, and acceleration. Additionally, the time step ΔT is introduced. In the typical scenarios, the lead vehicle is driving with variable acceleration; however, in order to make the scenarios even more difficult, sudden brakes are added. The host

² <https://www.fhwa.dot.gov/publications/research/operations/its/06135/>

vehicle is positioned behind the leading vehicle, with initial randomly generated velocity and acceleration.

Table 2. Parameters.

Parameter	$N[-]$	$\Delta T[s]$	$v[m/s]$	$a[m/s^2]$	$j[m/s^3]$	$\Delta s[m]$
Value	3	0.1	[0,60]	[-10,10]	[-2,2]	[0,10]

The parameter values used for all considered scenarios are shown in Tab. 2.

Safe MPC-based ACC. We analyze the simulation results by using two different deceleration profiles: full deceleration and mixed deceleration. For both cases, we evaluate the arithmetic mean \bar{j} , $\overline{\Delta s}$, \bar{d} , and the standard deviation σ_j , $\sigma_{\Delta s}$, σ_d associated with the variables j , Δs , and d for all considered vehicles, which are presented in Tab. 3.

Table 3. Simulation results.

Brake	$\bar{j}[m/s^3]$	$\sigma_j[m/s^3]$	$\overline{\Delta s}[m]$	$\sigma_{\Delta s}[m]$	$\bar{d}[m]$	$\sigma_d[m]$
Full	-0.005	0.883	3.369	3.557	22.073	11.244
Mixed	-0.006	0.298	0.287	1.071	23.773	6.080

Although the mean jerk value generated by applying full deceleration is small, the standard deviation shows that there is a broader range of jerk values, as can be seen in Tab. 3. Moreover, because of the frequent full braking, the safe distance tracking parameter, determined by $\overline{\Delta s}$, shows less performance, compared with the case when mixed deceleration is applied. For mixed deceleration, it can be seen that \bar{j} is small, which indicates comfortable driving without jerky maneuvers. The standard deviation σ_j is also small; thus most of the jerk values are close to the mean value. The results show good tracking performance, as the mean value of Δs is small. The average distance between vehicles is around 23m, which is comparable to the distance provided by the two-seconds distance rule³ (Martinez and de Wit, 2007), considering that the average velocity is 10.72m/s.

For illustration purposes, we only present the detailed simulation results for one considered scenario, whose duration is more than 2 minutes. The simulation results when applying the full deceleration profile are depicted in Fig. 5. It can be seen that big variations in the host vehicle's acceleration lead to big variations in velocity (see Fig. 5). Moreover, the jerk caused by often applying full braking results in uncomfortable driving.

Platooning using safe MPC-based ACC. To validate the string stability, a four-vehicle setup is considered as follows: The trajectory of vehicle #1 is taken from the US101 dataset; the other vehicles are placed behind one another, and they are controlled by our proposed algorithm. The task is that vehicle #2 safely follows vehicle #1, vehicle #3 follows vehicle #2, and vehicle #4 follows vehicle #3.

³ http://www.rotr.ie/rules-for-driving/speed-limits/speed-limits_2-second-rule.html

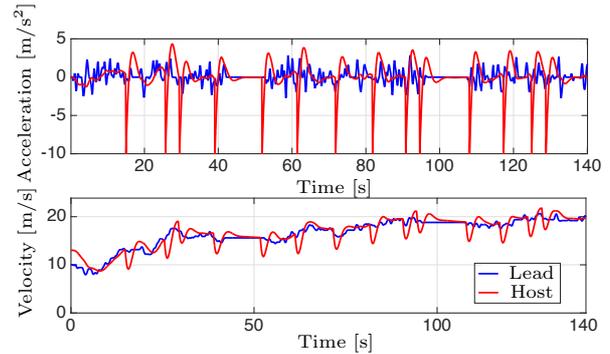


Fig. 5. Safe MPC-based ACC: Full deceleration when applying the emergency controller.

Here, we choose the mixed deceleration profile when applying the emergency controller, and it can be seen that the ACC-equipped vehicles (#2, #3, #4) smoothly follow the leading vehicle for the entire considered time; the velocity of the ACC-equipped vehicles also smoothly follows the velocity of the leading vehicle (see Fig. 6).

In order to not violate the safe distance, the safety mechanism is engaged for considered vehicles #2, #3, and #4 in 10.14%, 8.84%, and 8.63% of the time, respectively. However, the jerk value is kept between the specified comfortable value range (Hoberock, 1976). While the lead vehicle suddenly performs full braking, the ACC-equipped vehicles smoothly decelerate. Additionally, the position error Δs introduced by the leading vehicle braking is attenuated in the upstream direction, as illustrated in Fig. 6.

The mean jerk values \bar{j} and the standard deviation σ_j are small (see Tab. 3), which implies comfortable driving without jerky maneuvers. Keeping the inter-vehicle distance as close as possible to the safe distance d_{safe} by minimizing Δs shows good tracking performance. In this way, both safety and comfort are achieved by utilizing the proposed ACC concept.

The simulations were performed on a machine with 2.2 GHz, Intel i7 processor, and 16 GB 1600 MHz DDR3 memory, in Matlab R2015a. For solving the QP problem, the *quadprog* function from the Optimization Toolbox⁴ is used. The mean value of the computation time is 0.08s; therefore, the approach is real-time capable.

PI-based ACC. Finally, we compare our method with a state-of-the-art ACC approach applied in the automotive industry (Corona and Schutter, 2008; Yanakiev and Kanellakopoulos, 1998), which utilizes proportional-integral control (PI). Here, we use an implementation based on (Yanakiev and Kanellakopoulos, 1998), where the desired inter-vehicle distance is a function of a constant spacing, a constant time headway, and the velocity of the leading vehicle. Of course, other spacing policies can be used, as proposed in the aforementioned papers (e.g.: variable time headway).

Although the algorithm performs well with respect to position and velocity tracking, the PI controller itself cannot guarantee safety. Therefore, the controller fails to safely track the desired inter-vehicle distance (i.e. the

⁴ https://www.mathworks.com/help/pdf_doc/optim/optim_tb.pdf

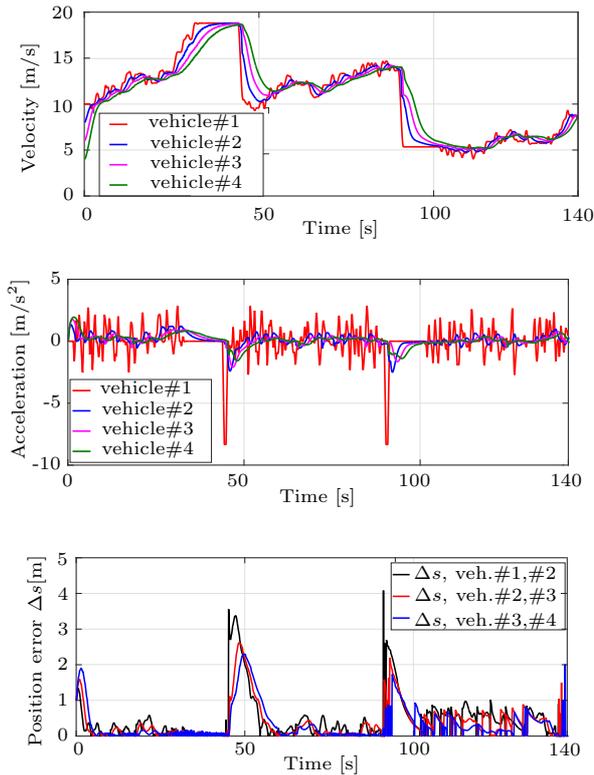


Fig. 6. Platooning using safe MPC-based ACC: Mixed deceleration when applying the emergency controller.

position errors Δs have negative values) as can be seen in Fig. 7.

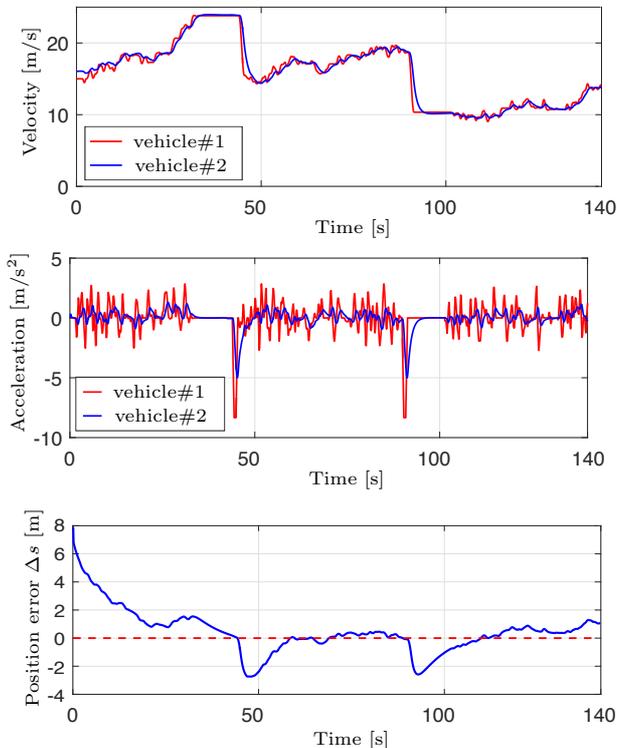


Fig. 7. Simulation results. PI-based ACC.

6. CONCLUSION AND FUTURE WORK

In this paper an ACC architecture consisting of an emergency and a nominal controller is designed in order to ensure safety and comfort. Safety is ensured by switching between the nominal controller and the emergency controller. The nominal controller is based on MPC, and it computes optimal inputs such that the safe distance is intended to be kept. In the emergency controller, the braking distance is solved analytically and computed based on the deceleration profile of the host vehicle, considering at each time step that the leading vehicle can fully brake. Moreover, the emergency deceleration profile is computed such that jerk values remain in the specified comfortable range. The proposed algorithm is evaluated using real traffic data, and it shows good performance on position and velocity tracking for all considered vehicles. Thus, we can conclude that our approach guarantees safety and comfort for ACC-equipped vehicles, which can take over the driving duties completely.

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