Technische Universität München
Department of Computer Sciene
I6 - Robotics & Embedded Systems
Dr. Reinhard Lafrenz
WS 2012/13

Exercises for the Lecture

Techniques in Artificial Intelligence

4.2.2013 – Sheet 4- Solutions

1) Markov processes I

Show that each second-order Markov process can be re-written as a first-order Markov process with a modified set of state variables. Does the number of parameters in the transition model increase?

For each variable U_t which is a parent of X_{t+2}, define a variable U_{t+1}^{old} as a parent of X_{t+2}. This variable "copies" the probabilities to the new auxiliary variable U_{t+1}^{old}, i.e. $P(U_{t+1}^{old}|U_t)$ keeps the values with a probability of 1. The CPT of X_{t+2} remains the same, only the name of the variable is changed.

All parameters are known and fixed, therefore the number of parameters do not increase.

2) Decision trees I

Consider the following table of 3 binary input attributes and one binary output.

Example	A1	A2	A3	Output y
x1	1	0	0	0
x2	1	0	1	0
x3	0	1	0	0
x4	1	1	1	1
x5	1	1	0	1

Construct a decision tree using these data.

Calculate the entropy of the problem, counting positive and negative examples.

H(<p/(p+n), n/(p+n)>) = H(<2/5, 3/5>) = <-2/5 ld(2/5), 3/5 ld(3/5) > = 0.528 + 0.442 = 0.97 Bits of information

The remainders for A1, A2, and A3 are

A1: (4/5) (-2/4 ld(2/4) – 2/4 ld(2/4)) + (1/5)(-0 -1/1ld(1/1)) = 0.800

A2: (3/5) (-2/3 ld(2/3) – 1/3 ld(1/3)) + (2/5)(-0 – 2/2 ld (2/2)) = 0.551

A3: (2/5) (-1/2) ld(1/2) – 1/2 ld (1/2)) + (3/5) (-1/3) ld(1/3) – 2/3) ld(2/3)) = 0.951

Now calc. the information gain, i.e. 0.97 – Remainder(A_i):

A1: 0.17

A2: 0.419

A3: 0.019

Therefore, A2 is chosen as the first attribute to test. It is now the root node

All attributes with A2=0 are already correctly classified, now consider the cases where A2=1 (x3, x4, x5).

Compute remaining information for A1 and A3

A1: (2/3)(-2/2 ld(2/2) -0) + (1/3) (-0 -1/1 ld(1/1)) = 0

A3: (1/3)(-1/1 ld(1/1) -0) + (2/3) (-1/2ld(1/2) – 1/2) ld(1/2)) = 0.667

3) Decision trees II

Develop a decision tree for the following data assuming you want to decide whether to play tennis based on the weather forecast in the morning.

Day	Outlook	Temperature	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

REVISED:

Entropy H(<9/14, 5/14>) = -9/14*ld(9/14) – 5/14*ld(5/14) = 0.94

Remainder(Outlook) = (5/14)*(-(2/5)*LOG(2/5;2) - (3/5)*LOG(3/5;2))
+ (4/14)*0 + (5/14)*(-(2/5)*LOG(2/5;2) - (3/5)*LOG(3/5;2)) =0.694

Remainder(Temp) = (4/14)*(-2/4*LOG(2/4;2) - 2/4*LOG(2/4;2))
+ (6/14)*(-4/6*LOG(4/6;2) - 2/6*LOG(2/6;2))
+ (4/14)*(-3/4*LOG(3/4;2) - 1/4*LOG(1/4;2)) = 0.911

Remainder (Humidity) =(7/14)*(-4/7*LOG(4/7;2) - 3/7*LOG(3/7;2))
+ (7/14)*(-6/7*LOG(6/7;2) -1/7*LOG(1/7;2)) =0.788

Remainder(Wind) =(8/14)*(-4/8*LOG(4/8;2) - 6/8*LOG(6/8;2))
+ (6/14)*(-3/6*LOG(3/6;2) -3/6*LOG(3/6;2))= 0.892

Information Gains: IG(Outlook) = 0.247, IG(Temperature) = 0.029, IG(Humidity) = 0.152, IG(Wind) = 0.048. Therefore, Outlook is chosen as the first attribute test. For this and the second level in case of outlook=sunny, the calculations are also given in the excel sheet which is on the web.