Vorlesung

Grundlagen der Künstlichen Intelligenz

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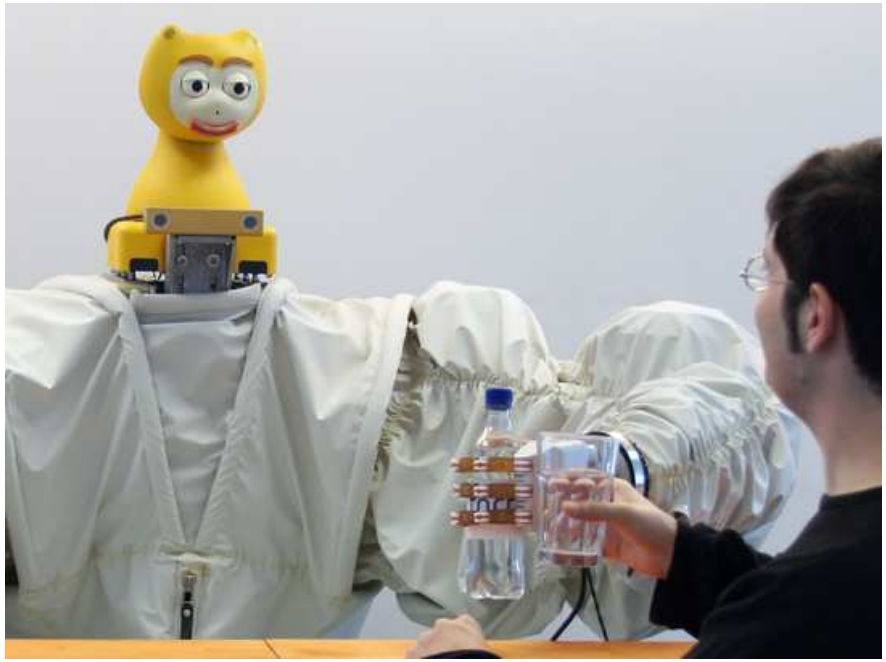

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Chapter 5 (3rd ed.)

Adversarial Search

Question:

- What is the difference between the 8-queen problem and playing chess?

- Reconsider the properties of different environments
  - Observability
  - Number of agents
  - Deterministic
  - etc.

Adversarial search

- Up to now: “Single-agent” problems
- Now: competitive environments, e.g. 2 agents competing
- Closely related to Game Theory

Adversarial search:

Typically 2-player zero-sum games

- Assumption: fully observable environments
- Examples: Chess, checkers

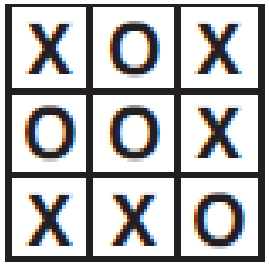
- Goal: Find optimal move (often with given time constraints)

Formal description of a game

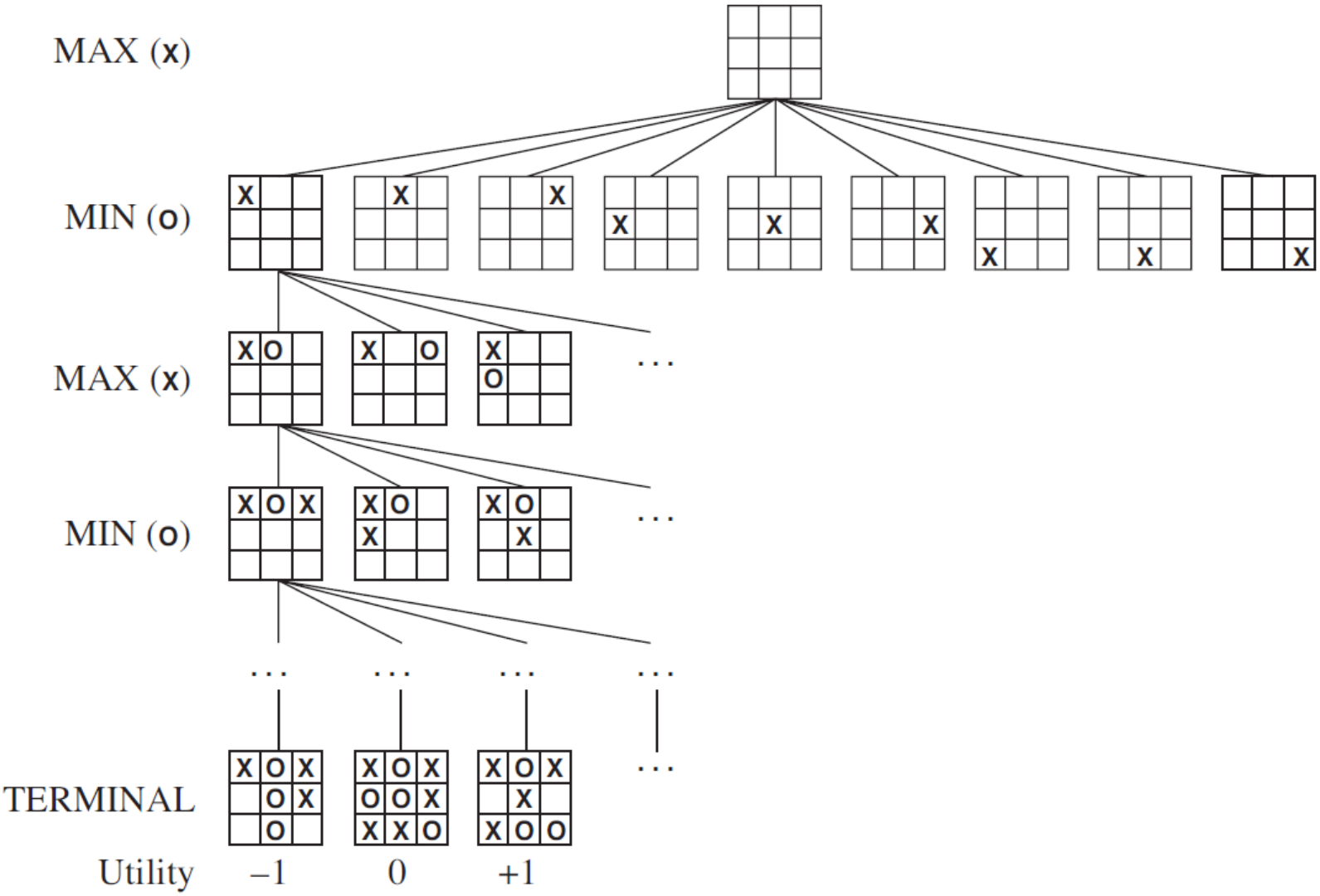
- S_0 the initial state
- PLAYER(*s*) player with next turn in a given state *s*
- ACTIONS (*s*) set of possible moves
- RESULT (*s*, *a*) transition model (result of a move)
- TERMINAL-TEST(*s*) true, if “game over“, false otherwise
- UTILITY (*s*, *p*) utility for a player *p* if game ends in state *s*

S_0, ACTIONS (*s*), RESULT (*s*, *a*) define a search tree

Simple example: Tic Tac Toe



Tic Tac Toe search tree: 2 players, MIN and MAX



Optimal decisions

Difference to classical search:

- Find a strategy, i.e. a sequence of optimal moves, taking the possible moves by the opponent into account

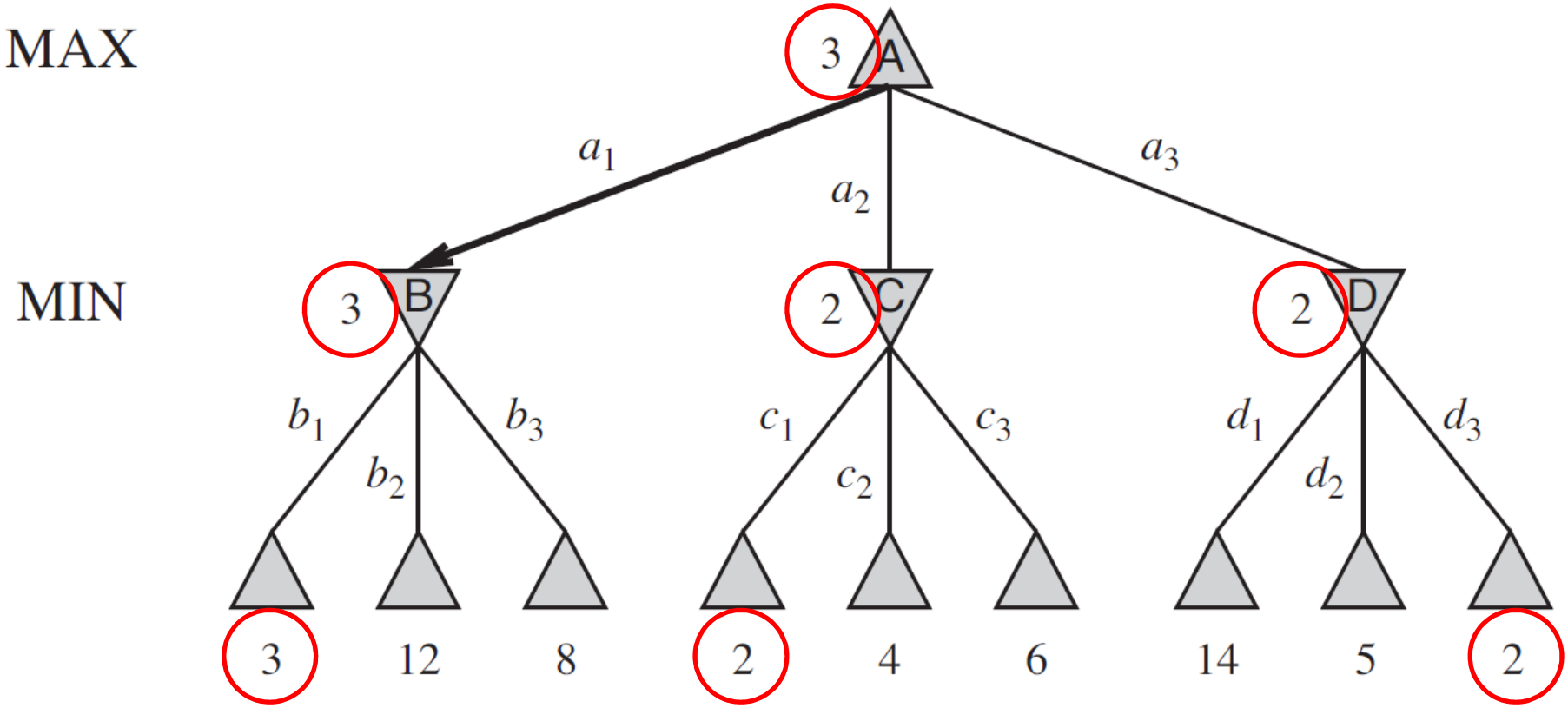
Definition of Minimax-function

$$\text{MINIMAX}(s) = \begin{cases} \text{UTILITY}(s) & \text{if TERMINAL-TEST}(s) = \text{true} \\ \max_{a \in \text{ACTIONS}(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s)=\text{MAX} \\ \min_{a \in \text{ACTIONS}(s)} \text{MINIMAX}(\text{RESULT}(s,a)) & \text{if PLAYER}(s)=\text{MIN} \end{cases}$$

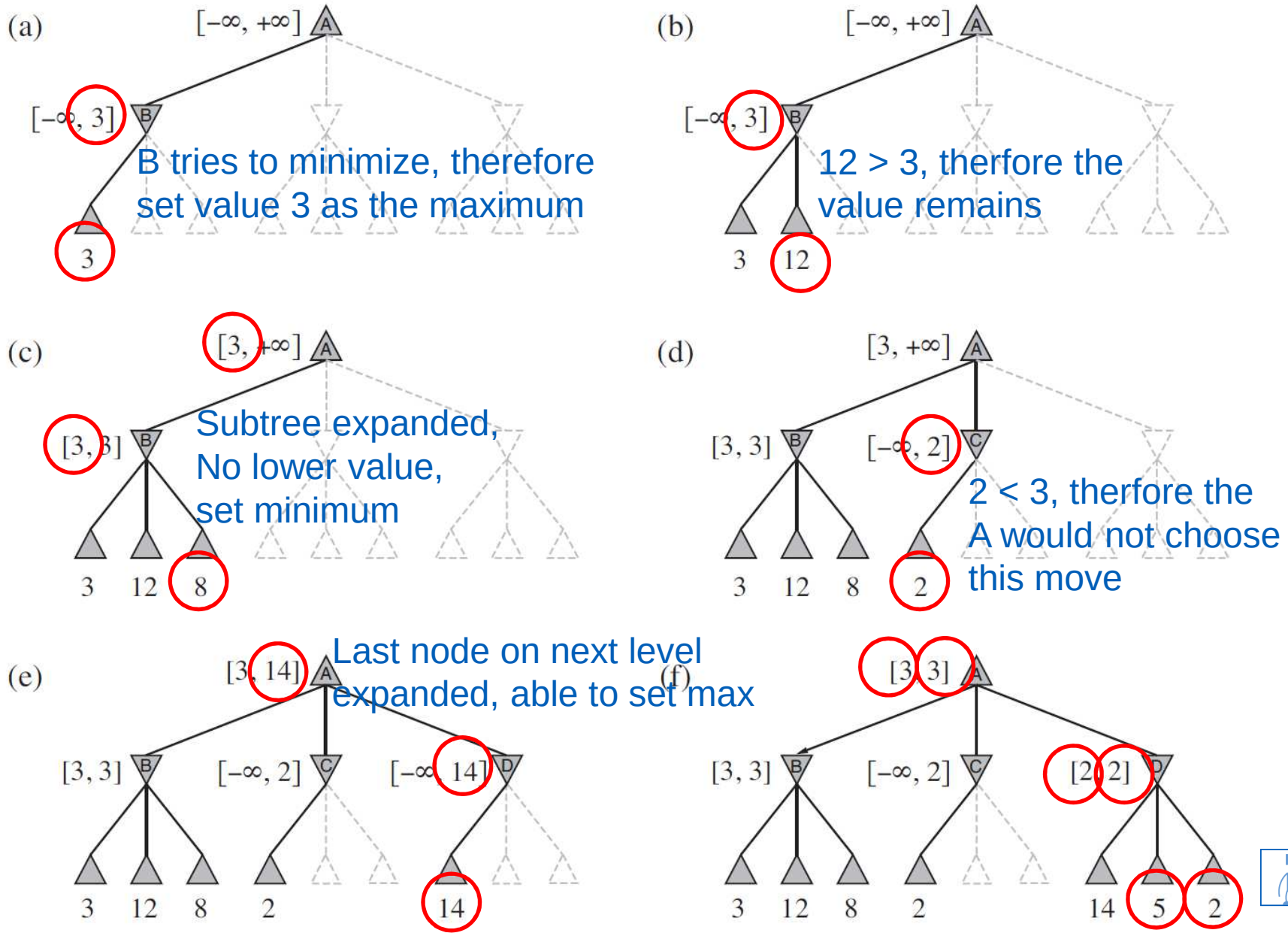
In a leaf(=final state of the game), Minimax(s) is the utility, otherwise the best value for the given player

Optimal decisions

Propagation of MIXIMAX values from the leaves to the root



Alpha-Beta Pruning



Alpha-Beta Pruning

function MINIMAX-DECISON(*state*) **returns** an action

return $\text{argmax}_{a \in \text{ACTIONS}(s)}$ MIN-VALUE(RESULT(*state*,*a*))

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function MAX-VALUE(*state*) **returns** an utility-value

  if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

  $v \leftarrow -\infty$

  foreach *a* **in** ACTIONS(*state*) **do**

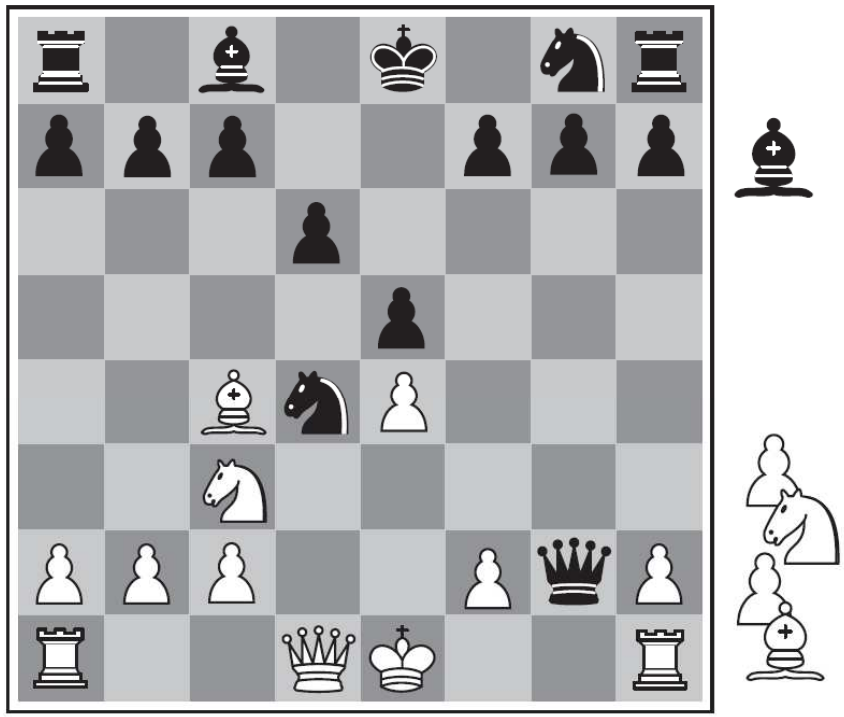
    $v \leftarrow$ MAX(v, MIN-VALUE(RESULT(*state*,*a*)))

  return *v*

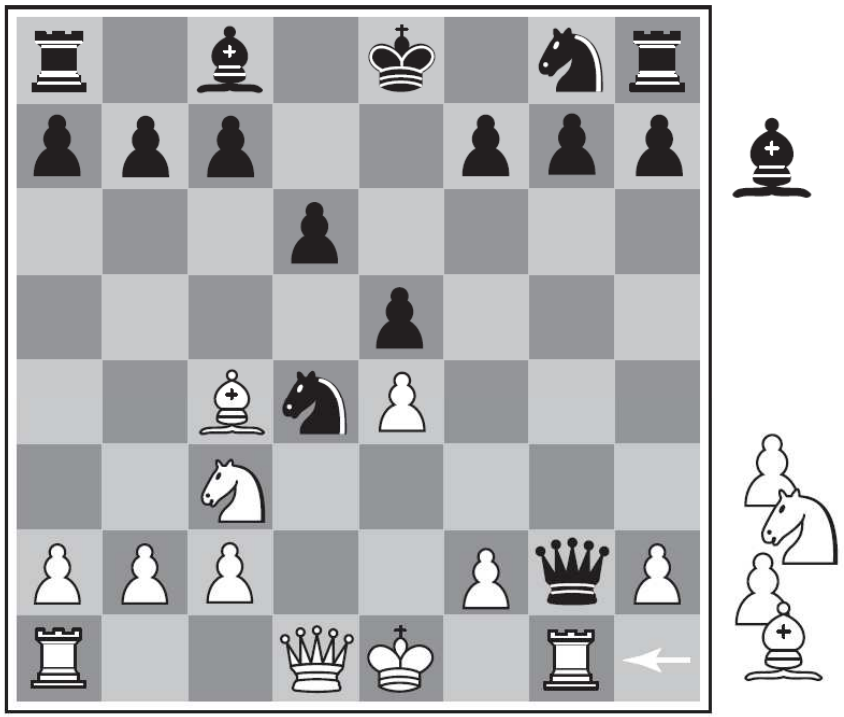
---

function MIN-VALUE(*state*) <same structure>

Incomplete real-time decisions



(a) White to move



(b) White to move

Incomplete real-time decisions

H-MINIMAX(s,d) =

$$\begin{cases} \text{EVAL}(s) & \text{if CUTOFF-TEST}(s,d) = \text{true} \\ \max_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s,a),d+1), & \text{if PLAYER}(s)=\text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s,a),d+1), & \text{if PLAYER}(s)=\text{MIN} \end{cases}$$

Evaluation function:

- Expectation values for UTILITY based on statistical data
  - Too many categories, not manageable
- Assumptions, e.g. „value“ of chessmen
  - Pawn: 1, Knight/bishop: 3, rook: 5, queen: 9
- Weighted linear function

Summary

- Adversarial search needs to take all possible moves of the opponent into account
- Maximise your strategy based on the assumpion that the opponent acts optimally

- Alpha-betra pruning can reduce the search space

- Incomplete real-time decisions need evaluation functions

Questions

Consider the travelling salesman problem (TSP)

- Minimal spanning tree (MST) heuristic: If partial tour already exists, MST of a subset of cities is the minimal sum of the connection cost of any ***tree*** connecting these cities

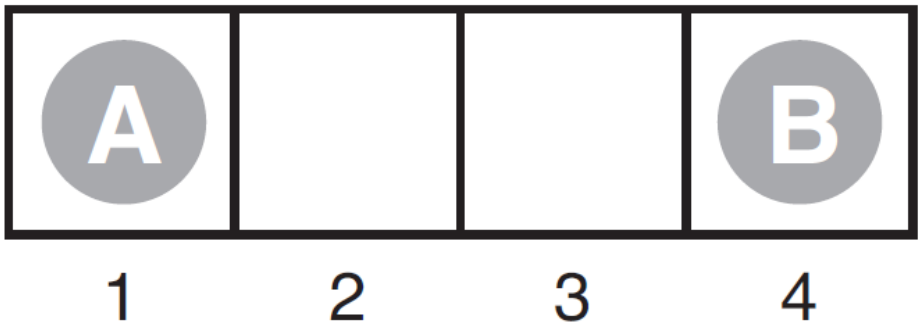
a) Show that this heuristic can be derived from a relaxed version of the TSP

b) Show that this heuristic dominates the straight-line distance

c) Are there efficient ways to solve the MST problem?

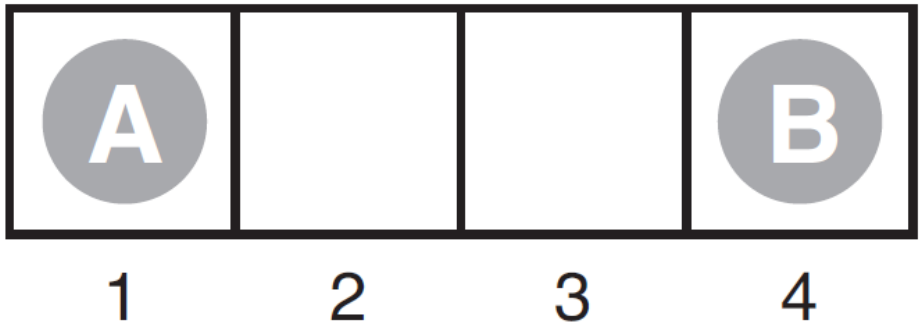
Questions

Consider the following game



- A moves first
- Each player must move to emtpy field in either direction
- If field occupied, the player can jump over the opponent, if there is a field
- Goal: reach the start field of the opponent
- Utility function for A: +1, if A reaches field 4 first
  - 1, if B reaches field 1 first

Questions



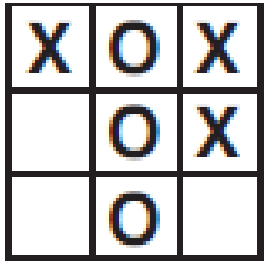
a) Draw the game tree using state (field for A, field for B)
   - terminal states drawn as squares, utility values in a circle
   - loop states drawn as double squares, their values are “?“

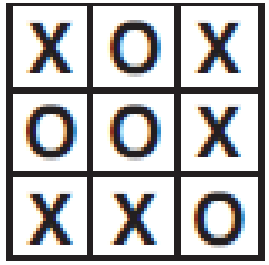
b) Mark the nodes with their MINIMAX values
   How are the “?“ handled? Why?

c) Why does the Minimax-algorithms fail for that tree?
   How can the tree be repaired?

Questions

a) How many Tic Tac Toe games are possible?





b) Draw a tree with 2 levels and show the utility values at level 2

c) Use the minimax algorithm to choose the best initial move

d) Show the nodes ***not*** expanded because of pruning-